Ren. NC QFT III: Progress in solving a Model in 4 dimensions
Kyoto, 23 rd February 2011

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Faculty of Physics, University of Vienna

(joint work with Raimar Wulkenhaar)

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Abstract

We study self-dual ϕ^4-theory on 4D Moyal space.

1. model is perturbatively renormalisable H G, Wulkenhaar
2. β-function vanishes to all orders Disertori-Gurau-Magnen-Rivasseau

The key for $\beta = 0$ are Ward identities and Schwinger-Dyson equations, but only the singular part is used.

- We extend this to a self-consistent non-perturbative integral equation for the renormalised two-point function alone.
- It can be solved perturbatively, without need of Feynman graphs, BPHZ renormalisation, forest formula etc.
- The solution takes values in a polynom ring. It is generated by iterated integrals labelled by rooted trees. The integrals evaluate to zeta functions and polylogarithms.

Standard Model

- As a classical field theory, it is a NC geometry.
- It gives rise to a perturbatively renormalised QFT.
- Scattering amplitudes are formal power series in coupling constants such as $e^2 \approx \frac{1}{137}$. Agreement with experiments.
- The radius of convergence in e^2 is zero!
- Refined summation techniques (e.g. Borel) may establish reasonable domains of analyticity.
- Unfortunately, this fails for QED: Landau ghost problem.
- May work for non-Abelian gauge theories due to asymptotic freedom, confinement not understood
- QFT's on noncommutative geometries may provide toy models for non-perturbative renormalisation in $D = 4$.

Matrix model

- Action in matrix base at $\Omega = 1$
- Action functionals for *bare* mass μ_{bare}
- Wave function renormalisation $\phi \mapsto Z^{\frac{1}{2}}\phi$.

Fix $\theta = 4$, $\phi_{mn} = \overline{\phi_{nm}}$ real:

$$S = \sum_{m,n \in \mathbb{N}^2_\Lambda} \frac{1}{2}\phi_{mn} H_{mn} \phi_{nm} + V(\phi)\,,$$

$$H_{mn} = Z\left(\mu^2_{bare} + |m| + |n|\right)\,, \qquad V(\phi) = \frac{Z^2\lambda}{4} \sum_{m,n,k,l \in \mathbb{N}^2_\Lambda} \phi_{mn}\phi_{nk}\phi_{kl}\phi_{lm}\,,$$

- Λ is cut-off. μ_{bare}, Z divergent
- No infinite renormalisation of coupling constant

$m, n, \ldots$ belong to $\mathbb{N}^2$, $|m| := m_1 + m_2$.

Ward identity

- inner automorphism $\phi \mapsto U\phi U^\dagger$ of M_Λ, infinitesimally $\phi_{mn} \mapsto \phi_{mn} + \mathrm{i}\sum_{k\in\mathbb{N}^2_\Lambda}(B_{mk}\phi_{kn} - \phi_{mk}B_{kn})$
- not a symmetry of the action, but invariance of measure $\mathcal{D}\phi = \prod_{m,n\in\mathbb{N}^2_\Lambda} d\phi_{mn}$ gives

$$
\begin{aligned}
0 &= \frac{\delta W}{\mathrm{i}\delta B_{ab}} = \frac{1}{\mathcal{Z}}\int \mathcal{D}\phi\,\Big(-\frac{\delta S}{\mathrm{i}\delta B_{ab}} + \frac{\delta}{\mathrm{i}\delta B_{ab}}(\mathrm{tr}(\phi J))\Big)e^{-S+\mathrm{tr}(\phi J)} \\
&= \frac{1}{\mathcal{Z}}\int \mathcal{D}\phi \sum_n \Big((H_{nb} - H_{an})\phi_{bn}\phi_{na} + (\phi_{bn}J_{na} - J_{bn}\phi_{na})\Big)e^{-S+\mathrm{tr}(\phi J)}
\end{aligned}
$$

where $W[J] = \ln \mathcal{Z}[J]$ generates connected functions

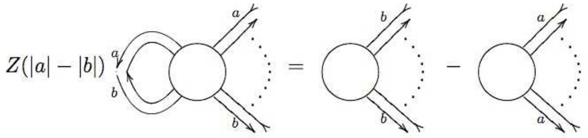
trick $\phi_{mn} \mapsto \frac{\partial}{\partial J_{nm}}$

$$
\begin{aligned}
0 = \Big\{ \sum_n \Big(&(H_{nb} - H_{an})\frac{\delta^2}{\delta J_{nb}\,\delta J_{an}} + \Big(J_{na}\frac{\delta}{\delta J_{nb}} - J_{bn}\frac{\delta}{\delta J_{an}}\Big)\Big) \\
&\times \exp\Big(-V\big(\tfrac{\delta}{\delta J}\big)\Big)e^{\frac{1}{2}\sum_{p,q} J_{pq}H^{-1}_{pq}J_{qp}}\Big\}_c
\end{aligned}
$$

Interpretation

The insertion of a special vertex $V^{ins}_{ab} := \sum_n (H_{an} - H_{nb})\phi_{bn}\phi_{na}$

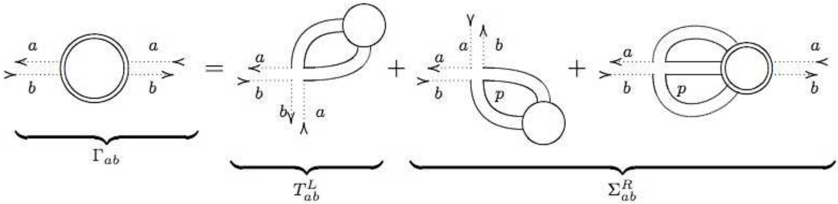
into an external face of a ribbon graph is the same as the difference between the exchanges of external sources $J_{nb} \mapsto J_{na}$ and $J_{an} \mapsto J_{bn}$



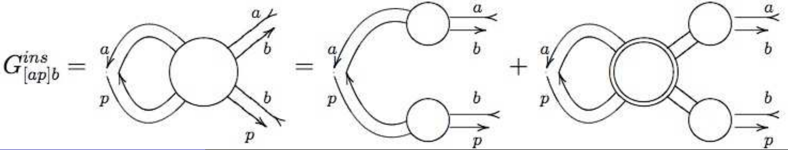
The dots stand for the remaining face indices.

$$Z(|a| - |b|)G^{ins}_{[ab]\dots} = \quad G_{b\dots} \quad - \quad G_{a\dots}$$

SD equation 2



- vertex is $Z^2\lambda$, connected two-point function is G_{ab}: first graph equals $Z^2\lambda \sum_q G_{aq}$
- open p-face in Σ^R and compare with insertion into connected two-point function; insert either into 1P reducible line or into 1PI function:



Amputate upper G_{ab} two-point function, sum over p, multiply by vertex $Z^2\lambda$, obtain: Σ^R_{ab}:

$$\Sigma^R_{ab} = Z^2\lambda \sum_p (G_{ab})^{-1} G^{ins}_{[ap]b} = -Z\lambda \sum_p (G_{ab})^{-1} \frac{G_{bp} - G_{ba}}{|p| - |a|} .$$

case $a = b = 0$ and $Z = 1$ treated.
Use $G^{-1}_{ab} = H_{ab} - \Gamma_{ab}$ and $T^L_{ab} = Z^2\lambda \sum_q G_{aq}$
gives for 2 point function:

$$Z^2\lambda \sum_q G_{aq} - Z\lambda \sum_p (G_{ab})^{-1} \frac{G_{bp} - G_{ba}}{|p| - |a|} = H_{ab} - G^{-1}_{ab} .$$

Symmetry $\Gamma_{ab} = \Gamma_{ba}$ is not manifest!

Renormalize

express SD equation in terms of the 1PI function Γ_{ab}
perform renormalisation for the 1PI part

$$\Gamma_{ab} = Z^2 \lambda \sum_p \Big(\frac{1}{H_{bp} - \Gamma_{bp}} + \frac{1}{H_{ap} - \Gamma_{ap}} - \frac{1}{H_{bp} - \Gamma_{bp}} \frac{(\Gamma_{bp} - \Gamma_{ab})}{Z(|p| - |a|)} \Big) \ .$$

Taylor expand:

$$\Gamma_{ab} = Z \mu^2_{bare} - \mu^2_{ren} + (Z-1)(|a|+|b|) + \Gamma^{ren}_{ab} \ ,$$
$$\Gamma^{ren}_{00} = 0 \qquad (\partial \Gamma^{ren})_{00} = 0 \ ,$$

$\partial \Gamma^{ren}$ is derivative in a_1, a_2, b_1, b_2. Implies

$$G^{-1}_{ab} = |a| + |b| + \mu^2_{ren} - \Gamma^{ren}_{ab} \ .$$

... the resulting equation is

$$(Z-1)(|a|+|b|) + \Gamma^{ren}_{ab} = \int_0^\Lambda |p|\, d|p| \Big(\frac{Z}{|b| + |p| + \mu^2 - \Gamma^{ren}_{bp}} + \frac{Z^2}{|a| + |p| + \mu^2 - \Gamma^{ren}_{ap}}$$
$$- \frac{Z^2 + Z}{|p| + \mu^2 - \Gamma ren_{0p}} \qquad - \frac{Z}{|b| + |p| + \mu^2 - \Gamma^{ren}_{bp}} \frac{\Gamma^{ren}_{bp} - \Gamma^{ren}_{ab}}{(|p| - |a|)} + \frac{Z}{|p| + \mu^2 - \Gamma^{ren}_{0p}} \frac{\Gamma^{ren}_{0p}}{|p|} \Big) \ ,$$

change variables,....

$$|a| =: \mu^2 \frac{\alpha}{1-\alpha}\ , \quad |b| =: \mu^2 \frac{\beta}{1-\beta}\ , \quad |p| =: \mu^2 \frac{\rho}{1-\rho}\ , \quad |p|\,d|p| = \mu^4 \frac{\rho\, d\rho}{(1-\rho)^3}$$
$$\Gamma_{ab} =: \mu^2 \frac{\Gamma_{\alpha\beta}}{(1-\alpha)(1-\beta)}\ , \quad \Lambda =: \mu^2 \frac{\xi}{1-\xi}$$

...get expression for ren.constant,

$$Z^{-1} = 1 + \lambda \int_0^{\xi} d\rho \frac{G_{0\rho}}{1-\rho} - \lambda \int_0^{\xi} d\rho \Big(G_{0\rho} - \frac{G'_{0\rho}}{1-\rho} \Big)$$

....Since the model is renormalisable, the limit $\xi \to 1$ can be taken...:

Results

....Since the model is renormalisable, the limit $\xi \to 1$ can be taken...:

Theorem

The renormalised planar two-point function $G_{\alpha\beta}$ of self-dual noncommutative ϕ^4_4-theory (with continuous indices) satisfies the integral equation

$$
\begin{aligned}
G_{\alpha\beta} = 1 + \lambda\Big(& \frac{1-\alpha}{1-\alpha\beta}(\mathcal{M}_\beta - \mathcal{L}_\beta - \beta\mathcal{Y}) + \frac{1-\beta}{1-\alpha\beta}(\mathcal{M}_\alpha - \mathcal{L}_\alpha - \alpha\mathcal{Y}) \\
& + \frac{1-\beta}{1-\alpha\beta}\Big(\frac{G_{\alpha\beta}}{G_{0\alpha}} - 1\Big)(\mathcal{M}_\alpha - \mathcal{L}_\alpha + \alpha\mathcal{N}_{\alpha 0}) - \frac{\alpha(1-\beta)}{1-\alpha\beta}(\mathcal{L}_\beta + \mathcal{N}_{\alpha\beta} - \mathcal{N}_{\alpha 0}) \\
& + \frac{(1-\alpha)(1-\beta)}{1-\alpha\beta}(G_{\alpha\beta} - 1)\mathcal{Y}\Big) ,
\end{aligned}
$$

$$
\mathcal{L}_\alpha := \int_0^1 d\rho\, \frac{G_{\alpha\rho} - G_{0\rho}}{1-\rho} , \qquad \mathcal{M}_\alpha := \int_0^1 d\rho\, \frac{\alpha\, G_{\alpha\rho}}{1-\alpha\rho} , \qquad \mathcal{N}_{\alpha\beta} := \int_0^1 d\rho\, \frac{G_{\rho\beta} - G_{\alpha\beta}}{\rho - \alpha}
$$

$\mathcal{Y} = \lim_{\alpha\to 0} \frac{\mathcal{M}_\alpha - \mathcal{L}_\alpha}{\alpha}$ *at the self-duality point.*

expansion

- Integral equation for Γ_{ab} is non-perturbatively defined. Resisted an exact treatment.
- We look for an iterative solution $G_{\alpha\beta} = \sum_{n=0}^{\infty} \lambda^n G^{(n)}_{\alpha\beta}$.
- This involves iterated integrals labelled by rooted trees.

Up to $\mathcal{O}(\lambda^3)$ we need

$$I_{\alpha} := \int_0^1 dx \; \frac{\alpha}{1-\alpha x} = -\ln(1-\alpha) \;,$$

$$I_{\underset{\bullet}{\alpha}} := \int_0^1 dx \; \frac{\alpha \, I_x}{1-\alpha x} = \mathrm{Li}_2(\alpha) + \frac{1}{2}\big(\ln(1-\alpha)\big)^2$$

$$I_{\underset{\bullet\;\bullet}{\alpha}} = \int_0^1 dx \; \frac{\alpha \, I_x \cdot I_x}{1-\alpha x} = -2\,\mathrm{Li}_3\Big(-\frac{\alpha}{1-\alpha}\Big)$$

$$I_{\underset{\underset{\bullet}{\bullet}}{\alpha}} = \int_0^1 dx \; \frac{\alpha \, I_{\underset{\bullet}{x}}}{1-\alpha x} = -2\mathrm{Li}_3\Big(-\frac{\alpha}{1-\alpha}\Big) - 2\mathrm{Li}_3(\alpha) - \ln(1-\alpha)\zeta(2)$$
$$+ \ln(1-\alpha)\mathrm{Li}_2(\alpha) + \frac{1}{6}\big(\ln(1-\alpha)\big)^3$$

Observations

Polylogarithms and multiple zeta values appear in singular part of individual graphs of e.g. ϕ^4-theory (Broadhurst-Kreimer)
We encounter them for regular part of all graphs together

Conjecture

- $G_{\alpha\beta}$ takes values in a polynom ring with generators $A, B, \alpha, \beta, \{I_t\}$, where t is a rooted tree with root label α or β
- at order n the degree of A, B is $\leq n$,
  the degree of α, β is $\leq n$,
  the number of vertices in the forest is $\leq n$.

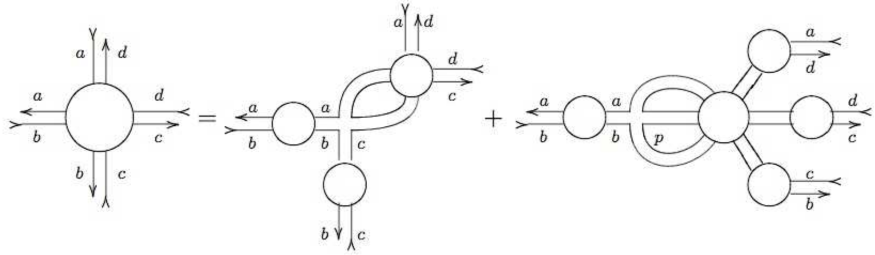
If true:

- There are $n!$ forests of rooted trees with n vertices at order n
- estimate: $|G^{(n)}_{\alpha\beta}| \leq n!(C_{\alpha\beta})^n$

may lead to Borel summability?.

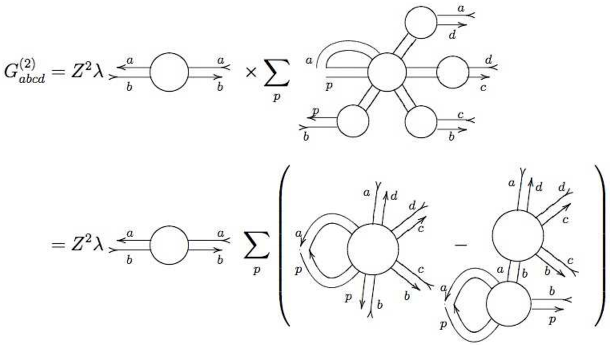
Schwinger-Dyson equ 4 pt fct

Follow *a*-face, there is a vertex at which *ab*-line starts:



1. First graph: Index c and a are opposite
   It equals $Z^2\lambda G_{ab}G_{bc}G^{ins}_{[ac]d}$
2. Second graph: Summation index p and a are opposite. We open the p-face to get an insertion.

   This is not into full connected four-point function, which would contain an *ab*-line not present in the graph.



second graph equals

$$= Z^2\lambda\Big(\sum_p G_{ab}G^{ins}_{[ap]bcd} - G^{ins}_{[ap]b}G_{abcd}\Big)$$

1PI four-point function

$$\Gamma^{ren}_{abcd} = Z\lambda\Big\{\frac{G^{-1}_{ad}-G^{-1}_{cd}}{|a|-|c|} + \sum_p \frac{G_{pb}}{|a|-|p|}\Big(\frac{G_{dp}}{G_{ad}}\Gamma^{ren}_{pbcd}-\Gamma^{ren}_{abcd}\Big)\Big\}$$

Theorem

The renormalised planar 1PI four-point function $\Gamma_{\alpha\beta\gamma\delta}$ of self-dual noncommutative ϕ^4_4-theory satisfies

$$\Gamma_{\alpha\beta\gamma\delta} = \lambda \cdot \frac{\left(1 - \frac{(1-\alpha)(1-\gamma\delta)(G_{\alpha\delta}-G_{\gamma\delta})}{G_{\gamma\delta}(1-\delta)(\alpha-\gamma)} + \int_0^1 \rho\, d\rho \frac{(1-\beta)(1-\alpha\delta)G_{\beta\rho}G_{\delta\rho}}{(1-\beta\rho)(1-\delta\rho)} \frac{\Gamma_{\rho\beta\gamma\delta}-\Gamma_{\alpha\beta\gamma\delta}}{\rho-\alpha}\right)}{G_{\alpha\delta}+\lambda\left((\mathcal{M}_\beta-\mathcal{L}_\beta-\mathcal{Y})G_{\alpha\delta}+\int_0^1 d\rho \frac{G_{\alpha\delta}G_{\beta\rho}(1-\beta)}{(1-\delta\rho)(1-\beta\rho)} + \int_0^1 \rho\, d\rho \frac{(1-\beta)(1-\alpha\delta)G_{\beta\rho}}{(1-\beta\rho)(1-\delta\rho)} \frac{(G_{\rho\delta}-G_{\alpha\delta})}{(\rho-\alpha)}\right)}$$

Corollary

$\Gamma_{\alpha\beta\gamma\delta} = 0$ is not a solution!
We have a non-trivial (interacting) QFT in four dimensions!

Conclusions

- Studied model at $\Omega = 1$
- RG flows save
- Used Ward identity and Schwinger-Dyson equation
- ren. 2 point fct fulfills nonlinear integral equ
- ren. 4 pt fct linear inhom. integral equ
  perturbative solution:

$$\Gamma_{\alpha\beta\gamma\delta} = \lambda - \lambda^2 \Big(\frac{(1-\gamma)(I_\alpha - \alpha) - (1-\alpha)(I_\gamma - \gamma)}{\alpha - \gamma} + \frac{(1-\delta)(I_\beta - \beta) - (1-\beta)(I_\delta - \delta)}{\beta - \delta} \Big) + \mathcal{O}(\lambda^3)$$

- is nontrivial and cyclic in the four indices
- nontrivial Φ^4 model ?

Spectral triples

see: A. Connes, "On the spectral characterization of manifolds,"

Definition (commutative spectral triple $(\mathcal{A}, \mathcal{H}, \mathcal{D})$ of dimension $p \in \mathbb{N}$)

...given by a Hilbert space $\mathcal{H}$, a commutative involutive unital algebra $\mathcal{A}$ represented in $\mathcal{H}$, and a selfadjoint operator $\mathcal{D}$ in $\mathcal{H}$ with compact resolvent, with

1. *Dimension:* k^{th} characteristic value of resolvent of $\mathcal{D}$ is $\mathcal{O}(k^{-\frac{1}{p}})$
2. *Order one:* $[[\mathcal{D}, f], g] = 0 \quad \forall f, g \in \mathcal{A}$
3. *Regularity:* f and $[\mathcal{D}, f]$ belong to domain of δ^k, where $\delta T := [|\mathcal{D}|, T]$
4. *Orientability:* $\exists$ Hochschild p-cycle $\boldsymbol{c}$ s.t.$\pi_{\mathcal{D}}(\boldsymbol{c}) = 1$ for p odd, $\pi_{\mathcal{D}}(\boldsymbol{c}) = \gamma$ for p even with $\gamma = \gamma^*$, $\gamma^2 = 1$, $\gamma\mathcal{D} = -\mathcal{D}\gamma$
5. *Finiteness and absolute continuity:* $\mathcal{H}_\infty := \cap_k \mathrm{dom}(\mathcal{D}^k) \subset \mathcal{H}$ is finitely generated projective $\mathcal{A}$-module, $\mathcal{H}_\infty = e\mathcal{A}^n$.

Spectral triples are interesting for physics!

- equivalence classes of spectral triples describe Yang-Mills theory (inner automorphisms; exist always in nc case) and possibly gravity (outer automorphisms)
- inner fluctuations: $\mathcal{D} \mapsto \mathcal{D}_A = \mathcal{D} + A\,, \quad A = \sum f[\mathcal{D}, g]$
  for almost-commutative manifolds: A=Yang-Mills+Higgs

Spectral action principle [Chamseddine+Connes, 1996]

As an automorphism-invariant object, the (bosonic) action functional of physics has to be a function of the spectrum of $\mathcal{D}_A$, i.e. $S(\mathcal{D}_A) = \mathrm{Tr}(\chi(\mathcal{D}_A))$.

for almost-commutative 4-dim compact manifolds:

- $S(\mathcal{D}_A) = \int_X d\,vol\ (\mathcal{L}_\Lambda + \mathcal{L}_{\text{EH+W}} + \mathcal{L}_{\text{YM}} + \mathcal{L}_{\text{Higgs-kin}} + \mathcal{L}_{\text{Higgs-pot}})$

  for any function of the spectrum (universality of RG)
- structure of the standard model more or less unique

U(1)-Higgs model for commutative algebra

tensor $(\mathcal{A},\mathcal{H},\mathcal{D}_1)$ with $(\mathbb{C}\oplus\mathbb{C},\mathbb{C}^2,M\sigma_1,\sigma_3)$ [Connes+Lott]

- $\mathcal{D}=\mathcal{D}_1\otimes\sigma_3+1\otimes\sigma_1 M=\begin{pmatrix}\mathcal{D}_1 & M\\ M & -\mathcal{D}_1\end{pmatrix}$ $\quad\begin{pmatrix} f & 0\\ 0 & g\end{pmatrix}\in\mathcal{A}_{tot}$
- selfadjoint fluctuated Dirac operators $\mathcal{D}_A:=\mathcal{D}+\sum_i a_i[\mathcal{D},b_i]$, $a_i,b_i\in\mathcal{A}_{tot}=\mathcal{A}\oplus\mathcal{A}$, are of the form
$$\mathcal{D}_A=\begin{pmatrix}\mathcal{D}_4+\mathrm{i}A^\mu\otimes(b^\dagger_\mu-b_\mu) & \phi\otimes 1\\ \overline{\phi}\otimes 1 & -(\mathcal{D}_4+\mathrm{i}B^\mu\otimes(b^\dagger_\mu-b_\mu))\end{pmatrix}$$
for $A_\mu=\overline{A_\mu},\ B_\mu=\overline{B_\mu},\ \phi\in\mathcal{A}$
- $D_\mu\phi=\partial_\mu\phi+i(A_\mu-B_\mu)\phi$
$$F_A=(-\{\partial^\mu,A_\mu\}-\mathrm{i}A^\mu A_\mu)\otimes 1+\tfrac{1}{4}F_A^{\mu\nu}\otimes[b^\dagger_\mu-b_\mu,b^\dagger_\nu-b_\nu]$$

Spectral action principle

most general form of bosonic action is $S(\mathcal{D}_A) = Tr\left(\chi(\mathcal{D}_A^2)\right)$

- Laplace transf. + asympt. exp'n $\mathrm{Tr}(e^{-t\mathcal{D}_A^2}) \sim \sum_{n=-dim/2}^{\infty} a_n(\mathcal{D}_A^2)t^n$

  leads to $S(\mathcal{D}_A) = \sum_{n=-dim/2}^{\infty} \chi_n \,\mathrm{Tr}(a_n(\mathcal{D}_A^2))$

  $$\textit{with} \quad \begin{array}{ll} \chi_z = \frac{1}{\Gamma(-z)} \int_0^\infty ds\, s^{-z-1}\chi(s) & \text{for } z \notin \mathbb{N} \\ \chi_k = (-1)^k \chi^{(k)}(0) & \text{for } k \in \mathbb{N} \end{array}$$

- a_n – Seeley coefficients, must be computed from scratch

Duhamel expansion: $\mathcal{D}_A^2 = \mathrm{H}_0 - V$

$$e^{-t(\mathrm{H}_0 - V)} = e^{-t\mathrm{H}_0} + \int_0^t dt_1 \left(e^{-(t-t_1)(\mathrm{H}_0 - V)} V e^{-t_1 \mathrm{H}_0}\right) \qquad \ldots\text{iteration}$$

Vacuum trace

Mehler kernel (in 4D)

$$e^{-t(H+\omega\Sigma)}(x,y) =$$
$$\frac{\omega^2(1-\tanh^2(\omega t))^2}{16\pi^2\tanh^2(\omega t)} e^{-t\omega\Sigma 1_2 - \frac{\omega}{4}\frac{\|x-y\|^2}{\tanh(\omega t)} - \frac{\omega}{4}\tanh(\omega t)\|x+y\|^2}$$
$$\mathrm{tr}(e^{-t\omega\Sigma}) = (2\cosh(\omega t))^d$$

$$\begin{aligned}\mathrm{Tr}(e^{-t(H+\omega\Sigma)\otimes 1_2}) &= 2\mathrm{tr}\Big(\int d^4x\ (e^{-t(\mathrm{H}+\omega\Sigma)})(x,x)\Big)\\ &= \frac{2}{\tanh^4(\omega t)} = 2(\omega t)^{-4} + \frac{8}{3}(\omega t)^{-2} + \frac{52}{45} + \mathcal{O}(t^2)\end{aligned}$$

- Spectral action is finite, in contrast to standard $\mathbb{R}^4$! (This is meant by “finite volume”)
- expansion starts with t^{-4} $\Rightarrow$ corresponds to 8-dim. space

The spectral action

$$S(\mathcal{D}_A) = \frac{2\chi_{-4}}{\omega^4} + \frac{8\chi_{-2}}{3\omega^2} + \frac{52\chi_0}{45} + \frac{\chi_0}{\pi^2}\int d^4x\ \Big\{ D^\mu\phi\overline{D_\mu\phi} + \frac{5}{12}(F^A_{\mu\nu}F_A^{\mu\nu} + F^B_{\mu\nu}F_B^{\mu\nu}) + \Big((|\phi|^2)^2 - \frac{2\chi_{-1}}{\chi_0}|\phi|^2 + 2\omega^2\|x\|^2|\phi|^2\Big)\Big\} + \mathcal{O}(\chi_1)$$

- spectral action is finite
- only difference in field equations to infinite volume is additional harmonic oscillator potential for the Higgs
- Yang-Mills is unchanged (in contrast to Moyal)
- vacuum is at $A_\mu = B_\mu = 0$ and (after gauge transformation) $\phi \in \mathbb{R}$, rotationally invariant

The spectral action: noncommutative case

$$S(\mathcal{D}_A) = \frac{\theta^4 \chi_{-4}}{8\Omega^4} + \frac{2\theta^2 \chi_{-2}}{3\Omega^2} + \frac{52\chi_0}{45} + \frac{\chi_0}{2\pi^2(1+\Omega^2)^2} \int d^4x \, \Big\{ 2 D_\mu \phi \star \overline{D_\mu \phi}$$
$$+ \Big(\frac{(1-\Omega^2)^2}{2} - \frac{(1-\Omega^2)^4}{3(1+\Omega^2)^2} \Big) \big(F^A_{\mu\nu} \star F_A^{\mu\nu} + F^B_{\mu\nu} \star F_B^{\mu\nu} \big)$$
$$+ \Big(\phi \star \bar{\phi} + \frac{4\Omega^2}{1+\Omega^2} X_A^\mu \star X_{A\mu} - \frac{\chi_{-1}}{\chi_0} \Big)^2$$
$$+ \Big(\bar{\phi} \star \phi + \frac{4\Omega^2}{1+\Omega^2} X_B^\mu \star X_{B\mu} - \frac{\chi_{-1}}{\chi_0} \Big)^2$$
$$-2 \Big(\frac{4\Omega^2}{1+\Omega^2} X_0^\mu \star X_{0\mu} - \frac{\chi_{-1}}{\chi_0} \Big)^2 \Big\}(x) + \mathcal{O}(\chi_1)$$

$$\Theta = \begin{pmatrix} 0 & \theta & 0 & 0 \\ -\theta & 0 & 0 & 0 \\ 0 & 0 & 0 & \theta \\ 0 & 0 & -\theta & 0 \end{pmatrix}$$

$$\omega = \frac{2\Omega}{\theta}$$

deeper entanglement of gauge and Higgs fields

covariant coordinates $X_{A_\mu}(x) = (\Theta^{-1})_{\mu\nu} x^\nu + A_\mu(x)$ appear with Higgs field ϕ in unified potential; vacuum is non-trivial!

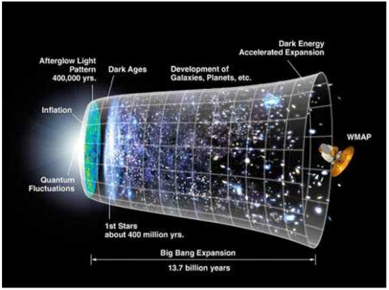
potential cannot be restricted to Higgs part if distinction into discrete and continuous geometries no longer possible

Outlook

Obtained renormalized II summable nc QFT. III locality weakened to Wedge locality

ren. sum. nc Standard Model?
ren. sum. Quantum Gravity?

- cosmological constant ???
- dark matter ???
- rotation curves of galaxies
- pionier10,11, MOND
- information paradox
- hierarchy problem
- divergences in QFT
- CONSISTENCY!!!???



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arigatou gozaimasu