Titles and abstracts of the talks of the

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and
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Paolo Bertozzini
Thammasat University

Non-commutative Geometries via Modular Theory

Abstract: We make use of Tomita-Takesaki modular theory in order to reconstruct non-commutative spectral geometries (formally similar to spectral triples) from suitable states over (categories of) operator algebras and further elaborate on the utility of such a formalism in an algebraic theory of quantum gravity, where space-time is spectrally reconstructed a posteriori from (partial) observables and states in a covariant quantum theory. Some relations with A. Carey-J. Phillips-A. Rennie modular spectral triples and with (loop) quantum gravity will be described.

Louis Boutet de Monvel
University of Paris 6

Asymptotics for Toeplitz Operators; residual trace, asymptotic equivariant index

Abstract: In these lectures I wish to give a review on Toeplitz operators (in the sense of Boutet de Monvel - Guillemin) and their relation to star products. I will describe the residual trace (essentially same as the 'Wodzicki residue') and the asymptotic equivariant trace and index, in presence of a compact group action, which is a useful extension. I wish to emphasize the case of complex spheres with a unitary torus action, which gives rise to a quite natural conjecture about the generators of the relevant K-theory, for which I only know the precise answer in special cases (one of which was described by Atiyah & Singer in their 1974 book).

Peter Bouwknegt
Australian National University

An introduction to T-duality

Abstract: In these lectures I will introduce the basics of T-duality, both from a physical and mathematical point of view. The lectures will focus on both differential geometric and algebraic topological aspects of T-duality. Connections to noncommutative geometry will be discussed in the follow-up lectures by Mathai Varghese.

Alan Carey
Australian National University

Noncommutative Nonunital Index Theory

Abstract: This talk is about work in progress with Gayral, Rennie and Sukochev motivated by index theory on non-compact manifolds, in particular, theorems of Gromov-Lawson type. The idea we are considering is whether there is a good notion of spectral triple for nonunital algebras that will lead to new index theorems on noncompact manifolds via a nonunital version of the local index formula in noncommutative geometry. Our aim is to avoid the use of local units.

In this talk I will
(i) explain how to define index pairings in terms of spectral triples in the nonunital situation.
(ii) describe an extension of the local index formula in noncommutative geometry to the nonunital case.
(iii) explain how to allow for the case when the spectral triple is semifinite.

Pierre Cartier
I.H.E.S.

About a group of diffeographisms in renormalization theory

Abstract: We shall review the well-known graphical representation of linear differential operators (in one or many variables) by trees . Using the Penrose graphical representation for tensors, we discuss the group of equivariant nonlinear transformations of tensors. In each case, there is a group, a Hopf algebra, a Vinberg algebra and a Lie algebra related to the group. We extend these constructions to the case of Feynman diagrams, with applications to the dimensional regularization and the Connes-Kreimer approach to renormalization.

Yakov Eliashberg
Stanford University

Symplectic invariants of affine complex manifolds via Legendrian surgery

Abstract: I will discuss computation of symplectic homology together with its product structure, and other invariants of Stein manifolds in terms of their symplectic handlebody decomposition. This is a joint work with F. Bourgeois and T. Ekholm.

Edward Frenkel
Berkeley

Langlands Program, Trace Formulas, and their Geometrization

Abstract: The Langlands Program relates Galois representations and automorphic representations of reductive algebraic groups defined over number fields and the fields of functions on curves over finite fields. The trace formula is a powerful tool in the study of this connection. After giving a brief introduction to the Langlands Program and its geometric version, which applies to curves over finite fields as well as over the complex field, I will outline a conjectural framework of "geometric trace formulas" in the case that the curve is defined over the complex field. It exploits a categorical formulation of the geometric Langlands correspondence. The talk is based on my joint work with Robert Langlands and Ngo Bao Chau (`arXiv:1003.4578`, `arXiv:1004.5323`).

Kenji Fukaya
Kyoto University

Homological mirror symmetry between toric A model and Landau-Ginzburg B model

Abstract: This is a joint work with M. Abousaid, Y.-.G. Oh, H. Ohta, K.Ono.

I will explain a proof of Homological mirror symmetry between toric A model and Landau-Ginzburg B model. More precisely I will explain the following. (The terminology used in the statement below will also be explained in the talk.) For any toric manifold X, there are finitely many Lagrangian submanifolds L_i equipped with an element b_i of its first cohomology group, such that the set $\{(L_i, b_i); i = 1, ..., N\}$ splits generates $Fuk(X)$ in the following sense. For any (L, b) a pair of Lagrangian submanifold of X and a weak bounding chain b such that $HF(L, b)$ is non zero, it is Floer theoretically undistinguishable from a direct summand of a direct sum of finitely many of (L_i, b_i)'s. The Floer cohomology together with A infinity structure between $(L_i b_i)$'s are the same as those obtained from the matrix factorization of the potential function PO which is defined as a generating function of open-close Gromov witten invariant on X. The proof is a combination of the isomorphism between quantum comology and Jacobian ring proved in FOOO `arXiv:1009.1648` and a version of Abousaid's criterion in Abouzaid `arXiv:1001.4593` which will be adapted to our situation where the symplectic manifold involved is not exact.

Ezra Getzler
Northwestern University

The automorphism 3-groupoid of the 2-groupoid of Poisson brackets on a manifold

Abstract: In general, the automorphisms of a groupoid form a 2-group. What about automorphisms of 2-groupoids? In this talk, I show explain how a certain abelian 3-group related to the de Rham, complex acts on the 2-groupoid of Poisson brackets on a manifold M.

The construction of this action is an application of Lie theory for L_∞ algebras.

Alexander Givental
Berkeley

The Riemann-Roch Theorem in Quantum K-theory

Abstract: Holomorphic Euler characteristics of interesting vector bundles over moduli spaces of stable maps into a Kähler target space can be considered as K-theoretic analogues of Gromov-Witten invariants of this target space. In these lectures, we will explain how such K-theoretic Gromov-Witten invariants of genus 0 can be characterized in terms of cohomological Gromov-Witten theory.

In the first lecture, we will introduce the problem and describe the result, obtained by Tom Coates an the speaker several years ago, which expresses the so-called "fake" K-theoretic GW-invariants (which, comparing to the genuine ones, are negligent to orbifolding) in terms of cohomological ones.

In the second lecture, we will explain how the genuine K-theoretic genus-0 GW-invarinats are caracterized in terms of fake ones. Thus obtained Quantum Hirzebruch-Riemann-Roch Theorem is a joint result of the speaker with Valentin Tonita.

In quantum cohomology theory, the so-called "quantum cohomology D-modules" arise naturally as the consequence of basic properties of genus-0 Gromov-Witten invariants. In quantum K-theory, examples suggest that finite-difference counterparts of D-modules should play a similar role. However the origin of this structure in quantum K-theory has remained a mystery. In the third lecture, we will explain how the D_q-module structures in quantum K-theory are naturally constructed on the basis of the QHRR theorem.

Dmitry Kaledin
Steklov Mathematical Institute

Witt vectors as a polynomial functor

Abstract: For any commutative ring A, one can define the commutative ring $W(A)$ of "Witt vectors"; this is functorial in A and, among other things, gives a functorial way to lift rings from char p to char 0. About 15 years ago, a generalization of this construction to non-commutative rings was proposed by L. Hesselholt (although for a non-commutative A, $W(A)$ is only a group). I am going to describe another possible generalization; it is certainly inspired by Hesselholt's work but formally independent of it. In the first talk, I will concentrate on the usual Witt vectors, and in particular, I will explain an invariant construction of the product in $W(A)$ (somewhat unexpectedly, the construction uses the Tate residue and the well-known central extension of the Lie algebra of infinite matrices). In the second talk, I will introduce a group $W(V)$ for any vector space V, and I will try to show what structures it has when V is an associative algebra, and how it is related to Witt vectors. If time permits, I will also introduce a version of the Hochschild complex which can be thought of as a non-commutative generalization of the de Rham-Witt complex of Deligne and Illusie.

Masaki Kashiwara
Kyoto University

Codimension three Conjecture

Abstract: The ring $\mathcal{E}$ of micro-differential operators is defined on the projective cotangent bundle P^*X of a manifold X. The support of coherent $\mathcal{E}$-modules is involutive. It has been known that the extension of the morphism between holonomic $\mathcal{E}$-modules has a "uniqueness property" (resp. "unique existence property") if it is defined outside subvariety of codimension $n+1$ (resp. $n+2$) where $n = dimX$. Now we announce that the regular holonomic $\mathcal{E}$-module defined outside a subvariety of codimension $n+3$ can be extended to a regular holonomic $\mathcal{E}$-module.

It is a joint work with K. Vilonen.

Maxim Kontsevich
I.H.E.S.

Quantum and hyperkähler geometry of integrable systems

Abstract: TBA

Giovanni Landi
University of Trieste

Quantum groups, spectral geometry and gauge fields

Abstract: I shall give a description of the spectral geometry (via spectral triples) of quantum groups and quantum homogeneous spaces. I shall also describe gauge fields, in particular monopoles and Higgs fields coupled via equivariant dimensional reduction, on these quantum spaces.

Hyun Ho Lee
Seoul National University

On Moduli space of a quantum Heisenberg manifold

Abstract: Although Sooran Kang reignited a study of Yang-Mills functional on a quantum Heisenberg manifold in her thesis, there has been known little on moduli space. In this talk, we report a progess in this direction.

Yutaka Matsuo
University of Tokyo

Applications of Nambu bracket in M-theory

Abstract: In M-theory, the description of the symmetry for multiply overlapping branes has been a long-standing mystery. In 2007, Bagger and Lambert found that the appropriate gauge symmetry for multiple M2-branes can be described by so called Lie 3-algebra. It is a generalization of the Lie algebra where the commutator is generalized by totally anti-symmetric 3-commutator which was originally proposed by Nambu long ago. The discovery of Bagger-Lambert theory triggered intensive researches on Lie 3-algebra and has been applied to M-theory. In this talk, I explain the recent development of the relation between Lie 3-algebra and M-theory together with some account on the contributions from our group.

Ruben Minasian
IPT, Saclay

Generalized complex geometry and string vacua

Abstract: I'll review some applications of Generalized complex geometry to string theory compactifications.

Takuro Mochizuki
RIMS, Kyoto University

From Hodge toward Twistor

Abstract: Recently, a generalized Hodge structure has been studied from various viewpoints. In this talk, we will discuss how the theory of holonomic D-module is enriched with a generalized Hodge structure, called twistor structure.

One of our final goal is to establish a twistor version of the celebrated theory of mixed Hodge modules due to M. Saito. After giving a survey on the pure case, in which we have already obtained a satisfactory theory, we would like to explain what is known and what should be known toward the mixed case.

Ryszard Nest
University of Copenhagen

Index theorem for symplectic deformations of gerbes

Abstract: We construct the trace density morphism from Hochschild and the negative cyclic homology of a symplectic deformation of a gerbe to the de Rham cohomology, and prove the index formula in this context.

Hideki Omori
Science University of Tokyo

Deformation of expressions of algebra

Abstract: As elements of Weyl algebra are expressed in many ways, e.g. normal/anti-normal ordering, Weyl ordering, there are in general many ways to express elements in the same isomorphism class. In this talk, I will give several interesting examples.
(1) The simplest one is the deformation given by

$$f(w)*_{\tau}g(w) = \sum_{n\in\mathbb{Z}}(\frac{\tau}{2})^n\frac{1}{n!}\partial_w^n f(w)\partial_w^n g(w).$$

$(\mathbb{C}[w]; *_{\tau})$ is an algebra isomorphic to the ordinary polynomial algebra, but there are some interesting transcendental elements:
(a) If $\mathrm{Re}\,\tau > 0.$ then $\sum_{n=-\infty}^{\infty} e_*^{inw}$ converges to give an element relating to Jacobi's theta function.
(b) The $*$-exponential function $e_*^{zw_*^2}$ of $w_*^2 = w{*}w$ has a double branching singularity at $z = \tau^{-1}$. This gives naturally a "central extension" of the 1-dimensional complex Lie algebra.
(2) The case of Weyl algebra generated by u, v with $[u, v] = -i\hbar$:
(a) $*$-exponential functions $e_*^{z\frac{1}{i\hbar}(au^2+bv^2+c(u*v+v*u))}$ of quadratic forms generate a group-like object, which behaves as if a "nontrivial double covering group" of $SL(2,\mathbb{C})$.
(b) Depending on generic expression parameters, there are three πi-periodic one parameter subgroup of $e_*^{z\frac{1}{i\hbar}(u*v+v*u)}$. Each of these yields matrix representation.

Akifumi Sako
Kushiro National College of Technology

Deformation Quantization of $U(N)$ Gauge Theory in $\mathbb{R}^4$

Abstract: We overview recent developments of noncommutative $U(N)$ gauge theory in $\mathbb{R}^4$ from the view point of deformation quantization. In general, value of an action integral or instanton number of a gauge theory is deformed by deformation quantization, and they become singular in some cases. To discuss how action functionals and gauge connections are deformed, mainly we consider instantons. Noncommutative instanton solutions deformed smoothly from commutative instantons are constructed, and we analyze their nature. In this talk, their asymptotic behaviors, instanton numbers, Green's functions, zero modes of Dirac operators, ADHM equations, and so on are discussed.

Pierre Schapira (1)
University of Paris 6

Sheaf quantization of Hamiltonian isotopies and applications

Abstract: This is a joint work with S. Guillermou and M. Kashiwara.
Recently Tamarkin presented a new approach to symplectic topology based on the microlocal theory of sheaves. For that purpose he had to adapt this theory which relies on the homogeneous symplectic structure to the non homogeneous case. Here, we remain in the homogeneous symplectic setting and prove various results of non displaceability, including the conservation of Morse inequalities as well as some results specific to positive isotopies. The main tool is a theorem which asserts that any Hamiltonian isotopy admits a unique sheaf quantization.

Pierre Schapira (2)
University of Paris 6

Deformation Quantization modules

Abstract: This is a joint work with M. Kashiwara.

We shall give an overview of recent results on modules over deformation quantization algebroids on complex Poisson manifolds. We study in particular the composition of kernels (finiteness and duality), the functoriality of Hochschild classes and the link with the Riemann-Roch theorem. Finally, we have a glance to holonomic modules on symplectic manifolds.

Mathai Varghese (1)
University of Adelaide

T-duality and noncommutative geometry

Abstract: When spacetime is a principal torus bundle with background flux, and one tries to iterate T-duality for circle bundles (as proved with Bouwknegt and Evslin) one circle at a time, some of the circles tend to mysteriously disappear, thereby preventing one from obtaining a classical T-dual. In this lecture, I will talk about how the use of noncommutative geometry solves the problem of the missing T-duals. The T-dual in this case is a C^*-bundle over the base space with fibres noncommutative tori with varying noncommutative parameter in the base direction. This is joint work with J. Rosenberg.

Mathai Varghese (2)
University of Adelaide

T-duality and parametrized strict deformation quantization

Abstract: In this lecture, I will talk about a parametrised version of Rieffel's strict deformation quantization, as developed with K. Hannabuss. I will show that many of the noncommutative T-duals from the previous lecture, can be constructed explicitly as parametrised strict deformation quantization of certain continuous trace algebras over principal torus bundles. This is mainly joint work with K. Hannabuss.

Mathai Varghese (3)
University of Adelaide

A noncommutative sigma model

Abstract: The general construction of sigma-models in which both the space-time manifold and the two-dimensional string world-sheet are both made noncommutative will be covered in this lecture. In particular, the sigma model when the space-time manifold is a noncommutative torus bundle will be made more explicit. The sigma model is analysed in detail in the special case where both the space-time manifold and the two-dimensional string world-sheet are both noncommutative tori. In this situation, we are able to determine when maps between such noncommutative tori exist, to derive the Euler-Lagrange equations, to classify many of the critical points of the Lagrangian, and to study the associated partition function. This is joint work with J. Rosenberg

Chris Woodward
Rutgers University

Symplectic topology of quotients I: Gauged Floer theory and non-displaceability of toric moment fibers

Abstract: I will describe an alternative proof of Fukaya-Oh-Ohta-Ono's recent results on non-displaceability of toric moment fibers using a limit of Frauenfelder's gauged Floer theory. For example, I will discuss which tori in some open subsets of $\mathbb{C}^2$ are known to be displaceable or non-displaceable.

Chris Woodward
Rutgers University

Symplectic topology of quotients II: Gauged Gromov-Witten theory and flops

Abstract: I will try to develop gauged Gromov-Witten/Floer theory more systematically for the action of a connected reductive group on a smooth variety of X. I will review the introduction of moduli spaces by Mundet and Salamon, and discuss joint work with Gonzalez in which we associate to each value of the "vortex parameter" a homotopy trace on the equivariant Gromov-Witten theory of X. I will mention some results on how the invariants depend on the vortex parameter, and how they change under flops.

Chris Woodward
Rutgers University

Symplectic topology of quotients III: Quantum Kirwan morphism of CohFT's

Abstract: I will describe partial results of Givental, Gaio-Salamon, and Ziltener on how the gauged invariants relate to the invariants of the quotient. Then I will describe a theorem in progress with Venugopalan which allows an algebraic construction of a morphism of CohFT's for an arbitrary git quotient.

Satoshi Yamaguchi
Osaka University

Polynomial structure in topological string

Abstract: The topological string is a string theory whose target space is a 3-dimensional Calabi-Yau manifold. It was found that the higher genus partition functions can be written as polynomials of finite number of generators. I would like to explain how this structure appears from the point of view of the BCOV holomorphic anomaly equation. I will also review recent development of this topic.

Weiping Zhang
Chern Institute, Nankai University

Geometric quantization on non-compact manifolds

Abstract: In the first talk, we will outline the analytic approach to the Guillemin-Stermberg geometric quantization conjecture, which can be described by the slogan of "quantization commutes with reduction", developed by Y. Tian and W. Zhang. In the second talk, we will show how the method of Tian and Zhang can be extended to generalize the Guillemin-Sternberg conjecture to non-compact settings.