Pair production of Dirac particles in noncommutative $d + 1$ spacetime

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Introduction

Motivations

• The quantum gravity is one of more important problem in physics, which intertwines between two fundamental theories: general relativity (GR) and quantum mechanics (QM).
It appeares when we consider higher mass in a few volume, as black hole physics. At the Planck length scale $\lambda_p = \sqrt{\frac{G\hbar}{c^3}} \approx 1,6 \cdot 10^{-35}$m, our description of the nature with these two fundamental theories break down.
• In the last two decades, several approaches have been given to tackle this issue from which we can identify very promising theories such as string theory, noncommutative (NC) geometry, loop quantum gravity, group field theory $\cdots$

Noncommutative quantum field theory

• Noncommutative field theory (NCFT), arising from NC geometry, has been the subject of intense studies, owing to its importance in the description of quantum gravity phenomena.
More precisely, the concepts of noncommutativity in fundamental physics have deep motivations which originated from the fundamental properties of the Snyder spacetime [S. Snyder (1947); Alain Connes (1991), S.Doplicher et al (1995), M. Douglas et al (2001)].

Introduction

• One of the important implications of noncommutativity is the *Lorentz violation symmetry* in more than two dimensions spacetime, which, in part, modifies the dispersion relations. It led to new developments in quantum electrodynamics (QED) and Yang-Mills (YM) theories in the NC variable function versions [R. Jackiw (2003), A. F. Ferrari et al (2006), M. Raasakka et al (2010)].
• Also, the quantum Hall effect well illustrates the NC quantum mechanics of spacetime [F. G. Scholtz et al (2005), B. Harms and O. Micu (2006)].

The NC $\star$-product is obtained by replacing the ordinary product of functions by the Moyal star product defined as follows:

$$(f \star g)(x) = \mathbf{m}\left[e^{\frac{i}{2}\theta^{\mu\nu}\partial_\mu \otimes \partial_\nu} f(x) \otimes g(x)\right] \tag{1}$$

where $f, g \in C^\infty(\mathbb{R}^D)$, $\mathbf{m}(f \otimes g) = f \cdot g$; $\theta^{\mu\nu}$ stands for a skew-symmetric tensor characterizing the NC behaviour of the spacetime, and has the Planck's length square dimension, i.e. $[\theta] \equiv [\lambda_p^2]$.

Introduction

In NCFT, we use a NC star product obtained by replacing the ordinary product of functions by the Moyal $\star$-product (1). It provides the following commutation relation between the coordinate functions:

$$[x^\mu, f]_\star = i\theta^{\mu\nu}\partial_\nu f, \quad x^\mu \in \mathbb{R}^D, \quad \mu,\nu = 1,2,3\cdots D. \tag{2}$$

Noncommutative quantum electrodynamics

Recall that two main problems arise when one tries to implement the electromagnetism in a NC geometry: the loss of causality due to the appearance of derivative coupling in the Lagrangian density and, more fundamentally, the violation of Lorentz invariance exhibited by plane wave solutions [R. Jackiw (2001); G. Berrino et al (2003)]. For example, the spacetime rotation R_μ^ρ leads to

$$\left[(R_\mu^\rho x^\mu = y^\rho), (R_\nu^\sigma x^\nu = y^\sigma)\right] = iR_\mu^\rho R_\nu^\sigma \theta^{\mu\nu} \neq i\theta^{\rho\sigma}. \tag{3}$$

Introduction

Like in ordinary quantum mechanics, the NC coordinates satisfy the coordinate-coordinate version of the Heisenberg uncertainty relation, namely $\Delta x^{\mu}\Delta x^{\nu} \geq \theta$, and then makes the spacetime a quantum space. This idea leads to the concept of quantum gravity, since quantizing spacetime leads to quantizing gravity.

The Moyal space $\mathbb{R}^D_\theta$

Let us define $E = \{\hat{x}^{\mu}, \mu \in 1, 2, \cdots D\}$ and $\mathbb{C}\langle E\rangle$ the algebra generated by E. Let (θ) a $D \times D$ non-degenerate skew-symmetric matrix (which requires D even) and I the ideal of $\mathbb{C}\langle E\rangle$ generated by the elements $\mathcal{R}^{\mu\nu} := \hat{x}^{\mu}\hat{x}^{\nu} - \hat{x}^{\nu}\hat{x}^{\mu} - \imath\theta^{\mu\nu}$. The Moyal algebra $\mathcal{A}_\theta$ is the quotient $\mathbb{C}\langle E\rangle / I$.

The very useful property of Moyal star product is :

$$\int d^D x (f \star g)(x) = \int d^D x f(x).g(x) = \int d^D x (g \star f)(x). \tag{4}$$

The Moyal algebra can be also defined as the linear space of smooth and rapidly decreasing functions equipped with the NC star product (1) in the form $f\star g = m[(\star_\theta \star_\hbar)(f\otimes g)]$ with

$$(f\star_\theta g) = m\left[\exp\left(\frac{i}{2}\theta^{ij}\partial_{x^i}\otimes\partial_{x^j}\right)(f\otimes g)\right] \quad (5)$$

$$(f\star_\hbar g) = m\left[\exp\left(\frac{i}{2}\hbar\delta^{ij}\left(\partial_{x^i}\otimes\partial_{p^j}-\partial_{p^i}\otimes\partial_{p^j}\right)(f\otimes g)\right)\right] \quad (6)$$

Scientific questions

Scientific question

How does the noncommutativity modifies the pair production of fermionic particles?

Plan

1. Preliminaries

2. Pair production of Dirac particles
   - Pair production of Dirac particles in ordinary spacetime
   - Pair production of Dirac particles in noncommutative spacetime

3. Concluding remarks

Plan

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   - Pair production of Dirac particles in ordinary spacetime
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3. Concluding remarks

Preliminaries

[J. Madore et al (2000)]

Let consider the spacetime geometry described with the NC coordinates x^μ and momentums p_μ, $\mu = 0, 1, 2, 3$, which satisfy the star-commutation relations :

$$[x^\mu, x^\nu]_\star = i\theta^{\mu\nu}, \quad [x^\mu, p_\mu]_\star = i\delta^\mu_\nu, \quad [p_\mu, p_\nu]_\star = 0, \tag{7}$$

in which, we shall take $\hbar := 1$ and where $\star$ denotes the Moyal star product (1),
and fields as elements of the algebra $\mathcal{A}_\theta$: $\psi(x) = \psi(x^1, x^2, x^3 x^4) \in \mathcal{A}_\theta$. We shall introduce the notion of an infinitesimal gauge transformation $\delta\psi$ of the field ψ and suppose that under an infinitesimal gauge transformation with the parameter $\alpha(x)$ it can be written in the form

$$\delta\psi(x) = i\alpha(x) \star \psi(x), \quad \alpha(x),\ \psi(x) \in \mathcal{A}_\theta. \tag{8}$$

Preliminaries

[N.Seiberg and E. Witten (1999), S. Fidanza (2002), K. Ulker et al (2012)]

Let's assume that the NC gauge field is written in the same function of commutative gauge and the NC gauge transformation parameter $\hat{\Lambda}$,

$$\hat{A}_\mu = (A_\mu;\theta), \quad \hat{F}_{\mu\nu} = \hat{F}_{\mu\nu}(A_\mu;\theta), \quad \hat{\Lambda} = \hat{\Lambda}_\alpha(\alpha, A_\mu;\theta) \tag{9}$$

Using the Seiberg-Witten maps at the first order of perturbation in θ, we write the NC field variables as function of commutative variables:

$$\hat{\psi} = \psi - \frac{1}{4}\theta^{\kappa\lambda}A_\kappa(\partial_\lambda + D_\lambda)\psi \tag{10}$$

$$\hat{\bar{\psi}} = \bar{\psi} - \frac{1}{4}\theta^{\kappa\lambda}A_\kappa(\partial_\lambda + D_\lambda)\bar{\psi} \tag{11}$$

$$\hat{A}_\mu = A_\mu - \frac{1}{4}\theta^{\kappa\lambda}\{A_\kappa, \partial_\lambda A_\mu + F_{\lambda\mu}\}. \tag{12}$$

Plan

Pair production of Dirac particles

[J. Schwinger (1948)]

Pair production is the creation of an elementary particle and its antiparticle, usually when a neutral boson interacts with a nucleus or another boson. This phenomenon is caused by the vacuum instability in the presence of electric fields near the critical strength value $E \approx Ec = m^2c^3/e\hbar \approx 1,3 \times 10^{16}\,V/cm$.

In the more original literature, there are two alternative ways for the computation of the probability density of the pair creaction but here, we presente only one.
It consists with the definition of the vacuum-vacuum transition amplitude Z and its complex norm which allows to derive the density $\omega(x)$ by the following formula:

$$|Z|^2 = \exp\left[-\int d^Dx\,\omega(x)\right] \tag{13}$$

This issue has been investigated in ordinary spacetime with the electromagnetic field by [Qiong-Gui Lin (1998), M. N. Hounkonnou and M. Naciri (2000)].

Plan

Pair production of Dirac particles in NC spacetime 16

The action for a Dirac particle on NC spacetime can be defined as follows:

$$S = \int_{\mathbb{R}^4} d^4x\, \mathcal{L}(\hat{\bar{\psi}}, \hat{\psi}), \quad \mathcal{L}(\hat{\bar{\psi}}, \hat{\psi}) = \hat{\bar{\psi}} \star i\gamma^\mu \hat{D}_\mu \hat{\psi} - m\hat{\bar{\psi}} \star \hat{\psi}, \tag{14}$$

where $\hat{\psi}$ and $\hat{\bar{\psi}}$ are the Dirac spinors and its associated Hermitian conjugate, respectively. The γ's are the Dirac matrices which satisfy the Clifford algebra: $\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$, and are given explicitly in terms of Pauli matrices σ^i, $i = 1, 2, 3$, by:

$$\gamma^0 = \begin{pmatrix} 1_2 & 0 \\ 0 & -1_2 \end{pmatrix}, \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}. \tag{15}$$

The covariant derivative $\hat{D}_\mu$ is expressed as: $\hat{D}_\mu = \partial_\mu - i\hat{A}_\mu \star$. We choose $\hbar = c = 1$ and take the charge of particle equal to the unit value, i.e. $q_e = 1$.

Pair production of Dirac particles in NC spacetime 17

By substituting the expressions of fields $\hat{\psi}$, $\hat{\bar{\psi}}$, $\hat{A}_\mu$ obtained by solving Seiberg-Witten equations in (10),(11),(12) in the action (14), we get, at the first order in θ the following Lagrangian density:

$$\begin{aligned}\mathcal{L}(\bar{\psi},\psi) &= i\gamma^\mu\Big[\bar{\psi}(\partial_\mu - iA_\mu - m)\psi + \frac{i}{2}\theta^{\alpha\beta}\partial_\alpha\bar{\psi}\partial_\beta(\partial_\mu - iA_\mu)\psi - \frac{1}{4}\theta^{\alpha\beta}\bar{\psi}\partial_\mu\Big(A_\alpha(\partial_\beta + D_\beta)\psi\Big) \\ &+ \frac{1}{2}\theta^{\alpha\beta}\bar{\psi}\partial_\alpha A_\mu\partial_\beta\psi + \frac{i}{4}\theta^{\alpha\beta}\bar{\psi}A_\mu A_\alpha(\partial_\beta + D_\beta)\psi + \frac{i}{4}\theta^{\kappa\lambda}\bar{\psi}\{A_\kappa, \partial_\lambda A_\mu + F_{\lambda\mu}\}\psi \\ &- \frac{1}{4}\theta^{\kappa\lambda}A_\kappa(\partial_\lambda + D_\lambda)\bar{\psi}(\partial_\mu - iA_\mu)\psi\Big] - m\Big[\frac{i}{2}\theta^{\mu\nu}\partial_\mu\bar{\psi}\partial_\nu\psi - \frac{1}{4}\theta^{\mu\nu}\bar{\psi}A_\mu(\partial_\nu + D_\nu)(\psi) \\ &- \frac{1}{4}\theta^{\mu\nu}A_\mu(\partial_\nu + D_\nu)(\bar{\psi})\psi\Big] + o(\theta^2). \end{aligned} \quad (16)$$

In the commutative limit where $\theta \to 0$, we recover to the Lagrangian density $\mathcal{L}_C$. The action $\mathcal{S}[\psi,\bar{\psi},A]$ in (14) can be write as the form

$$\mathcal{S}[\psi,\bar{\psi},A] = \int d^4x\,\mathcal{L}(\bar{\psi},\psi) \approx \int d^4x\,\Big(\mathcal{L}_C(\bar{\psi},\psi) + \mathcal{B}(\theta, A, \bar{\psi},\psi)\Big), \quad (17)$$

Pair production of Dirac particles in NC spacetime

where the quantity $\mathcal{B}(\theta, A, \bar{\psi}, \psi)$ depending on θ is given by

$$\begin{aligned}\mathcal{B}(\theta, A, \bar{\psi}, \psi) &= i\gamma^{\mu}\theta^{\kappa\lambda}\bar{\psi}\Big[-\frac{1}{2}(\partial_{\mu}A_{\kappa})\partial_{\lambda}+\frac{1}{2}\partial_{\kappa}A_{\mu}\partial_{\lambda}+\frac{i}{2}A_{\mu}A_{\kappa}\partial_{\lambda}+\frac{i}{2}A_{k}\partial_{\lambda}A_{\mu}-\frac{i}{2}A_{k}\partial_{\mu}A_{\lambda}\\ &+ \frac{1}{2}(\partial_{\lambda}A_{\kappa})\partial_{\mu}-\frac{i}{2}(\partial_{\lambda}A_{k})A_{\mu}\Big]\psi-\frac{m\theta^{\kappa\lambda}}{2}\bar{\psi}(\partial_{\kappa}A_{\lambda})\,\psi. \end{aligned} \tag{18}$$

Now by performing the path integral over the background fields ψ and $\bar{\psi}$, the vacuum-vacuum transition amplitude $\mathcal{Z}(A)$ is afforded by the expression:

$$\mathcal{Z}(A) = \mathcal{N}\int D\psi D\bar{\psi}\exp i\Big\{\int d^4x\,\Big(i\gamma^{\mu}\bar{\psi}(\partial_{\mu}-iA_{\mu})\psi - m\bar{\psi}\psi + \mathcal{B}(\theta, A, \bar{\psi}, \psi)\Big)\Big\}, \tag{19}$$

in which the normalization constant $\mathcal{N}$ is chosen such that $\mathcal{Z}(0) = 1$. Note that $\mathcal{B}(\theta, 0, 1, 1) = 0$.

Pair production of Dirac particles in NC spacetime 19

Let $\mathcal{M} := i\gamma^\mu D_\mu - m + \mathcal{B}(\theta, A, 1, 1) + i\varepsilon$. Then, we get a simpler form:

$$\mathcal{Z}(A) = \exp\left[-tr \ln \frac{i\gamma^\mu \partial_\mu - m + i\varepsilon}{\mathcal{M}}\right]. \tag{20}$$

Now, we consider the EM field, defined in x direction as $\mathbf{B} = B\mathbf{e_x}$ and $\mathbf{E} = E\mathbf{e_x}$, $E > 0$ and $B \geq 0$. The position and momentum operators $X_\mu = (X_0, X_1, X_2, X_3) =: (X_0, X, Y, Z)$ and $P_\mu = i\partial_\mu = (P_0, P_1, P_2, P_3)$ satisfy the commutation relation: $[X_\mu, P_\mu] = i\eta_{\mu\nu}$. The covariant vector V_μ is expressed with the contravariant V^μ as $V_\mu = \eta_{\mu\nu} V^\nu$, where $(\eta) = diag(1, -1, -1, -1)$. The covariant Faraday tensor $F_{\mu\nu} =: \partial_\mu A_\nu - \partial_\nu A_\mu$ with $A_\mu = (-EX, 0, 0, BY)$. Then, $\mathcal{B}(\theta, A, 1, 1)$ is obtained as:

$$\begin{aligned} \mathcal{B}(\theta, A, 1, 1) &= \frac{m\theta}{2}(B - E) + \frac{i\theta}{2}\gamma^\mu \Big[i(E + B)A_\mu - (E + B)\partial_\mu - iA_\mu(EX\partial_1 + BY\partial_2) \\ &- (\partial_1 A_\mu)\partial_0 + (\partial_2 A_\mu)\partial_3 + \partial_\mu(EX)\partial_1 + \partial_\mu(BY)\partial_2 \Big]. \end{aligned} \tag{21}$$

Pair production of Dirac particles in NC spacetime

Using the charge conjugation matrix $C = i\gamma^2\gamma^0$, the identity $C\gamma_\mu C^{-1} = -\gamma_\mu^t$, and taking into account the fact that the trace of an operator is invariant under a matrix transposition lead to

$$\mathcal{Z}^t(A) = \exp\left[-tr\ln\frac{iC\gamma^\mu C^{-1}\partial_\mu + m - i\varepsilon}{\mathcal{M}^t}\right], \tag{22}$$

where $\mathcal{M}^t = iC\gamma^\mu C^{-1}D_\mu + m - \mathcal{B}^t(\theta, A, 1, 1) - i\varepsilon$.

The probability density is defined by the module of $Z(A)$ as

$$|\mathcal{Z}(A)|^2 := \exp\left[-tr\ln\frac{P^2 - m^2 + i\varepsilon}{\mathcal{M}\mathcal{M}^t}\right], \tag{23}$$

with

$$\mathcal{M}\mathcal{M}^t = [\gamma^\mu(P_\mu + A_\mu)]^2 - m^2 - m^2\theta(B - E) + \mathcal{B}\gamma^\mu(P_\mu + A_\mu) - \gamma^\mu(P_\mu + A_\mu)\mathcal{B}^t + i\varepsilon. \tag{24}$$

Pair production of Dirac particles in NC spacetime

The conjugate of $\mathcal{B}(\theta, A, 1, 1)$, denoted by $\mathcal{B}^t(\theta, A, 1, 1)$ can be then written as:

$$\begin{aligned}\mathcal{B}^t(\theta, A, 1, 1) &= \frac{m\theta}{2}(B-E) + \frac{i\theta}{2}C\gamma^\mu C^{-1}\Big[i(E+B)A_\mu - iA_\mu(EX\partial_1 + BY\partial_2) \\ &- (E+B)\partial_\mu - (\partial_1 A_\mu)\partial_0 + (\partial_2 A_\mu)\partial_3 + \partial_\mu(EX)\partial_1 + \partial_\mu(BY)\partial_2\Big]. \qquad (25)\end{aligned}$$

At this point it would be worth using the identity

$$\ln\frac{a+i\varepsilon}{b+i\varepsilon} = \int_0^\infty \frac{ds}{s}\left[e^{is(b+i\varepsilon)} - e^{is(a+i\varepsilon)}\right] \qquad (26)$$

to get

$$\ln\frac{P^2 - m^2 + i\varepsilon}{\mathcal{M}\mathcal{M}^t} = \int_0^\infty \frac{ds}{s} e^{-is(m^2 - i\varepsilon)}\left[e^{is[(P+A)^2 + \frac{1}{2}\sigma^{\mu\nu}F_{\mu\nu} - m^2\theta(B-E) + \mathcal{X}(\theta)]} - e^{isP^2}\right] \qquad (27)$$

where the operator $\mathcal{X}(\theta)$ should be Hermitian.

Pair production of Dirac particles in NC spacetime

For an arbitrary operator $\mathcal{A}$ we can define the associated Hermitian operator denoted by $\mathcal{A}_H$ as $\mathcal{A}_H = \frac{(\mathcal{A}+\mathcal{A}^\dagger)}{2}$.

We then have the following:

Proposition

The Hermitian operator associated with $\mathcal{X}(\theta)$, denoted $\mathcal{X}_H(\theta)$, is given by

$$\begin{aligned} \mathcal{X}_H(\theta) &= \frac{\theta}{2}\Big[iEB\gamma^3\gamma^2 + iE^2\gamma^0\gamma^1 + iB^2\gamma^3\gamma^2 + \frac{1}{2}i(\gamma^0\gamma^1+\gamma^3\gamma^2)EB + \gamma^0\gamma^1 EBYP_2 \\ &+ 2E^2\gamma^0\gamma^1 XP_1 + \gamma^0\gamma^3(E^2B - EB^2)XY - \gamma^1\gamma^3 EBYP_1 - (4E^3+3BE^2)X^2 \\ &+ (2B^3+B^2E)Y^2 + (4E^2+5EB)XP_0 + (2B^2+3EB)YP_3 - 2BP_0^2 + 2BP_1^2 \\ &+ 2EP_2^2 + 2EP_3^2\Big]. \end{aligned} \tag{28}$$

$\mathcal{X}_H(\theta) = \mathcal{X}_H^\dagger(\theta)$.

Pair production of Dirac particles in NC spacetime 23

Now we focus on the computation of the following quantity:

$$\mathcal{Q} = \langle \mathbf{x}| e^{is[(P+A)^2+\frac{1}{2}\sigma^{\mu\nu}F_{\mu\nu}-m^2\theta(B-E)+\mathcal{X}_H(\theta)]}|\mathbf{x}\rangle \tag{29}$$

which can write as

$$\mathcal{Q} = \mathcal{Q}_c + \mathcal{Q}_{nc}(\theta), \quad \mathcal{Q}_{nc}(0) = 0 \tag{30}$$

where

$$\mathcal{Q}_c = e^{\frac{is}{2}\sigma^{\mu\nu}F_{\mu\nu}} \langle \mathbf{x}| e^{is(P+A)^2}|\mathbf{x}\rangle \tag{31}$$

and

$$\mathcal{Q}_{nc}(\theta) = e^{\frac{is}{2}\sigma^{\mu\nu}F_{\mu\nu}} \int d\mathbf{y}\, \langle \mathbf{x}| e^{is(P+A)^2}|\mathbf{y}\rangle \langle \mathbf{y}| is\left[m^2\theta(E-B)+\mathcal{X}_H(\theta)\right]|\mathbf{x}\rangle. \tag{32}$$

We then come to the following result:

Pair production of Dirac particles in NC spacetime 24

Theorem

Let $\theta =: \wp \cdot \theta_0$ where $\wp$ is a dimensionless quantity which is bounded by two numbers a_1, and a_2 and such that $\theta_0 << 1$. The mass dimension of θ_0 is obviously $\theta_0 \equiv [M^{-2}]$. Let $M \subset \mathbb{R}^2$ be the compact subset of $\mathbb{R}^2$ in which the following integral

$$\int_{M\subset\mathbb{R}^2} \frac{dt}{t_0} dz = b \equiv [M^{-2}]. \tag{33}$$

The trace of the expectation value $\mathcal{Q}$ is given by:

$$tr\mathcal{Q} = \Big(1 - \wp - \sigma(\theta_0, E, B)\Big) tr\mathcal{Q}_c, \tag{34}$$

$$\sigma(\theta_0, E, B) = \frac{16\pi^3 b \wp \exp\left[is\theta_0 \mathcal{G}_0\right]}{\theta_0^3 s^3 EB} \sqrt{\frac{B}{E} f(E,B)}, \qquad tr\mathcal{Q}_c = -\frac{1}{4\pi^2 i} EB\cosh(Es)\cot(Bs),$$

$$f(E,B) = \left[4B^6 + 76EB^5 + 258E^2B^4 + 494E^3B^3 + 224E^4B^2 + 12E^5B\right]^{-1}. \tag{35}$$

Pair production of Dirac particles in NC spacetime

Remark that the quantity $\sigma(\theta_0, E, B)$ leads to the divergence in the limit where $B = 0$ and in the limit where $\theta_0 = 0$. This expression do not contribute to the physical solution.

Theorem

The vacuum-vacuum transition probability is $|Z(A)|^2 = \exp\left[-\int dx\, \omega(x)\right]$ where

$$\omega(x) = \frac{1}{4\pi^2 i}\int_0^\infty ds\, \frac{e^{ism^2}}{s}\left[\left(1-\wp\right) EB\coth(Es)\cot(Bs) - \frac{1}{s^2}\right] \tag{36}$$

whose real part, denoted by $\Re_e\omega(x) = \frac{\omega+\omega^\star}{2}$, is given by

$$\begin{aligned}\Re_e\omega(x) &= -\frac{1}{8\pi^2 i}\int_{-\infty}^\infty ds\, \frac{e^{ism^2}}{s}\left[\left(1-\wp\right) EB\coth(Es)\cot(Bs) - \frac{1}{s^2}\right] \\ &= -\frac{m^4\wp}{16\pi} + \frac{EB}{4\pi^2}(1-\wp)\sum_{k=1}^{\infty}\frac{1}{k}\coth\left(k\pi\frac{B}{E}\right)\exp\left(-\frac{k\pi m^2}{E}\right).\end{aligned} \tag{37}$$

Pair production of Dirac particles in NC spacetime

Discussion

If $\Re_e\omega_c(x)$ is the probability density provided in the equation (37), in the limit where $\theta \to 0$ i.e.

$$\lim_{\theta\to 0} \Re_e\omega(x) = \Re_e\omega_c(x), \tag{38}$$

then this expression corresponds to the commutative limit derived by [Qiong-Gui Lin (1998), M. N. Hounkonnou and M. Naciri (2000)] and given by:

$$\Re_e\omega_c(x) = \frac{EB}{4\pi^2}\sum_{k=1}^{\infty}\frac{1}{k}\coth\left(k\pi\frac{B}{E}\right)\exp\left(-\frac{k\pi m^2}{E}\right). \tag{39}$$

We get

$$\Re_e\omega(x) < \Re_e\omega_c(x), \tag{40}$$

and we conclude that the noncommutativity increases the amplitude $|Z(A)|^2$. This shows the importance of noncommutativity in the high energy regime in which creation of particle is being manifested.

Concluding remarks

Concluding remarks

- In this presentation, we talk about the NC theory of fermionic field interacting with its corresponding boson. We have used the Seiberg-Witten expansion describing the relation between the NC and commutative variables, to compute the probability density of pair production of NC fermions. We have showed that, in the limit where the NC parameter $\theta = 0$, we recover the result of the previous litterature.
- Our study has highlighted that the noncommutativity of spacetime increases the density ω of the probability of pair creation of the fermion particle.

Thank you for your attention