Un voyage dans les univers de de Sitter autour des notions de système élémentaire aux sens classique (Kostant-Kirillov-Souriau) & quantique (Wigner, théorie quantique des champs)
Lecture 1:
The question of mass and rest energy in (Anti-) de Sitter space-times with regard to Minkowski

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Porto Novo, Bénin, 11-16 Juillet 2022

- Les espace-temps à symétrie maximale: Minkowski, de Sitter et anti-de-Sitter

Un panorama du groupe de Poincaré et de ses deux déformations, de Sitter et anti-de-Sitter, est présenté du point de vue de leurs duals (duaux?) unitaires respectifs. Des applications récentes de ces considérations de symétries en théorie quantique des champs et en cosmologie sont décrites

- Space-times with maximal symmetries: Minkowski, de Sitter et anti-de-Sitter

A survey of the Poincaré group and its two deformations, de Sitter and anti-de-Sitter, is presented from the viewpoint of their respective unitary duals. Recent applications of these symmetry approaches to quantum field theories and in cosmology are described

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The de Sitter Group and its Representations

An Introduction to Elementary Systems and Modeling the Dark Energy Universe

May 11, 2022

Springer Nature

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General considerations about the mass

- In Minkowski space-time, the concept of (rest or proper) mass or rest energy originates in the ubiquitous law of conservation of energy, a direct consequence of the Poincaré symmetry.
- As soon as we deal with de Sitter or Anti de Sitter space-time (i.e. background), this concept of mass or rest energy should be thoroughly reconsidered.
- In particular, one might expect to lose a precise distinction between “massive” and “massless”.
- So, we should look for other properties, e.g. existence or violation of conformal invariance, of some gauge invariance, in view of extending concepts about mass inherited from Minkowskian physics.

Elementary system in the Wigner sense

From T. D. Newton and E. P. Wigner, Localized States for Elementary Systems, *Rev. Mod. Phys.* 21, 400-406 (1949)

- *The concept of an "elementary system" requires that all states of the system be obtainable from the relativistic transforms of any state by superpositions. In other words, there must be no relativistically invariant distinction between the various states of the system which would allow for the principle of superposition. This condition is often referred to as irreducibility condition*

- *The concept of an elementary system (...) is a description of a set of states which forms, in mathematical language, an irreducible representation space for the inhomogeneous Lorentz ($\simeq$ Poincaré) group*

Wigner classification of Poincaré UIR's

From Wigner E.P., On Unitary Representations of the Inhomogeneous Lorentz Group, *Ann. Math.* 40, 149-204 (1939)

The unitary irreducible representations (UIR) of the Poincaré group are completely characterized by the eigenvalues of its two Casimir operators,

- **Quadratic** (Klein-Gordon operator)

$$Q^{(1)}_{\text{Poincaré}} = P^{\mu}\, P_{\mu} = P^{0\,2} - \mathbf{P}^2 \equiv P^2$$

(P^{μ} : translation generators) with eigenvalues

$$\langle Q^{(1)}_{\text{Poincaré}} \rangle = m^2\, c^2\,,$$

- **Quartic** (Pauli-Lubanski operator)

$$Q^{(2)}_{\text{Poincaré}} = W^{\mu} W_{\mu}\,,\ W_{\mu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} J^{\nu\rho} P^{\sigma}\,,$$

($J^{\nu\rho}$: 6 Lorentz generators) with eigenvalues (in the non-zero mass case)

$$\langle Q^{(2)}_{\text{Poincaré}} \rangle = -m^2\, c^2\, s(s+1)\hbar^2.$$

Wigner classification according to mass operator and the little group UIR's (0ptional)

First Casimir or squared mass	P^μ	Little group
(a) $P^2 = m^2c^2 > 0\,, P^0 > 0$	$(mc, 0, 0, 0)$	SO(3)
(b) $P^2 = m^2c^2 > 0\,, P^0 < 0$	$(-mc, 0, 0, 0)$	SO(3)
(c) $P^2 = 0\,, P^0 > 0$	$(\kappa, \kappa, 0, 0)$	ISO(2)
(d) $P^2 = 0\,, P^0 < 0$	$(-\kappa, \kappa, 0, 0)$	ISO(2)
(e) $P^2 = N^2 > 0$	$(0, N, 0, 0)$	SO$(2,1)$
(f) $P^\mu = 0$	$(0, 0, 0, 0)$	SO$(3,1)$

The only physical cases are respectively

(a) massive representations with positive energy, denoted $\mathcal{P}^{>}(m, s)$

(c) massless representations with positive energy, denoted $\mathcal{P}^{>}(0, s)$

(f) vacuum

Uniqueness of deformations of Poincaré kinematical symmetry

From H. Bacry and J. M. Levy-Leblond, Possible Kinematics, *J. Math. Phys.* 9 1605-1614 (1968)

- *With the requirements of kinematical rotation, parity, and time-reversal invariance, there exists only one way to "deform" the Poincaré group, namely, in endowing space-time with a certain curvature*

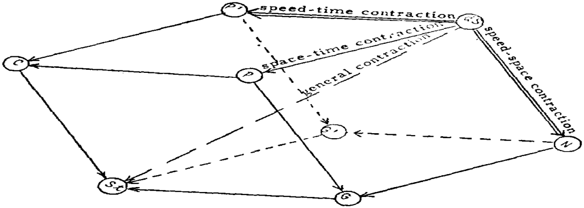


FIG. 1. The contraction scheme for the relativity groups.

Relative-time groups:	(*dS*),	(*P′*),	(*P*),	(*C*).
Absolute-time groups:	(*N*),	(*G′*),	(*G*),	(*St*).
Relative-space groups:	(*dS*),	(*N*),	(*P*),	(*G*).
Absolute-space groups:	(*P′*),	(*G′*),	(*C*),	(*St*).
Cosmological groups:	(*dS*),	(*N*),	(*P′*),	(*G′*).
Local groups:	(*P*),	(*G*),	(*C*),	(*St*).

Remark: The effect of the symmetry S of Eq. (6) is equivalent to a symmetry of the cube with respect to the plane containing the vertices (*dS*), (*N*), (*C*), *and* (*St*).

The question of mass in (Anti-) de Sitter quantum physics

- The classical context
  (i) Geometry
  (ii) Symmetries
- The quantum context
  (i) UIR's of de Sitter group
  (ii) UIR's of Anti de Sitter group
  (iii) Null-curvature limit
- Proper mass and rest energy in "(Anti) desitterian Physics"
- Dark matter as a relic AdS pure curvature energy?

General Relativity (GR) : Two distinct theories proposed by Einstein

There were elaborated by Einstein to deal respectively with local gravitational phenomena and within a cosmological context

Theory 1

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = -\kappa\, T_{\mu\nu} - \Lambda\, g_{\mu\nu}\,.$$

Here, the fundamental state that contains the maximum number of symmetries is the Minkowskian geometry.
$\Lambda > 0 \sim$ "dark energy"
$\Lambda < 0 \sim$ "dark matter"?

Theory 2

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda\, g_{\mu\nu} = -\kappa\, T_{\mu\nu}\,.$$

Here, the fundamental states that contain the maximum number of symmetries are the de-Sitter (dS) ($\Lambda \equiv \Lambda_{\rm dS} > 0$) and the Anti-de-Sitter (AdS) ($\Lambda \equiv \Lambda_{\rm AdS} < 0$) geometries.

General remarks on the interest of dS/AdS studies

- dS and AdS are *maximally symmetric*
  (in a metric space of dimension n, the maximum number of metric preserving symmetries is $n(n+1)/2$, here 10 since $n=4$)
- Their symmetries are one-parameter deformations of Minkowskian symmetry with
  - negative curvature $-\varkappa_{\mathrm{dS}} = -H/c = -\sqrt{\Lambda_{\mathrm{dS}}/3}$ (H : Hubble parameter)
  - positive curvature $\varkappa_{\mathrm{AdS}} = \sqrt{|\Lambda_{\mathrm{AdS}}|/3}$

  respectively
- As soon as a constant curvature is present (like the currently observed one), we lose some of our so familiar conservation laws like energy-momentum conservation!
- What is then the physical meaning of a scattering experiment ("space" in dS is like the sphere $\mathbb{S}^3$, let alone the fact that time is ambiguous)?
- Which relevant "physical" quantities are going to be considered as (asymptotically? contractively?) experimentally available?

dS perturbation of minkowskian background (optional)

Typical dimensionless parameter for dS perturbation of minkowskian background, $\vartheta \equiv \vartheta_m =: \frac{\hbar\sqrt{\Lambda}}{\sqrt{3}mc} = \frac{\hbar H}{mc^2} = \frac{\hbar\, \varkappa_{\mathrm{dS}}}{mc} \approx 0.293 \times 10^{-68} \times m_{\mathrm{kg}}^{-1}$ for some known (rest minkowskian) masses m and the present day estimated value of the Hubble radius

$$c/H_0 \approx 1.2 \times 10^{26}\mathrm{m},$$

(distance between the Earth and the galaxies which are currently receding from us at the speed of light)

Mass m	$\vartheta_m \approx$
$m_\Lambda/\sqrt{3} \approx 0.293 \times 10^{-68}$kg	1
up. lim. photon mass m_γ	0.29×10^{-16}
up. lim. neutrino mass m_ν	0.165×10^{-32}
electron mass m_e	0.3×10^{-37}
proton mass m_p	0.17×10^{-41}
$W^{\pm}$ boson mass	0.2×10^{-43}
Planck mass M_{Pl}	0.135×10^{-60}

We easily understand from this table that the currently estimated value of the cosmological constant has no practical effect on our familiar massive fermion or boson fields. Contrariwise, adopting the de Sitter point of view appears as inescapable when we deal with infinitely small masses, as is done in standard inflation scenario.

The symmetries

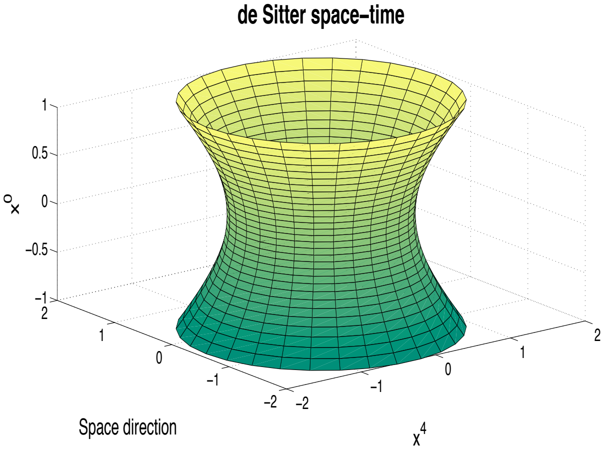
- De Sitter [resp. Anti-de Sitter] space-times are the unique solutions *with maximal symmetry* of the vacuum Einstein's equations with positive [resp. negative] cosmological constant Λ. This constant is linked to the (constant) Ricci curvature 4Λ of these space-times
- There exists a fundamental length $\ell_\Lambda := \sqrt{3/|\Lambda|}$ or equivalently a universal frequency ν_Λ or a universal curvature $\varkappa_\Lambda = \varkappa_{\mathrm{dS}}$ or $\varkappa_{\mathrm{AdS}}$
- Respective invariance (in the relativity or kinematical sense) groups : the ten-parameter de Sitter $\mathrm{SO}_0(1,4)$ (or $\mathrm{Sp}(2,2)$) and anti de Sitter $\mathrm{SO}_0(2,3)$ (or $\mathrm{Sp}(4,\mathbb{R})$) groups.
- Both are deformations of the proper orthochronous Poincaré group $\mathbb{R}^{1,3} \rtimes \mathrm{SO}_0(1,3)$ (or $\mathbb{R}^{1,3} \rtimes \mathrm{SL}(2,\mathbb{C})$), the kinematical group of Minkowski spacetime

de Sitter geometry

- de Sitter space can be viewed as a one-sheeted hyperboloid embedded in a five-dimensional Minkowski space

$$M_{dS} \equiv \{x \in \mathbb{R}^5;\ x^2 = \eta_{\alpha\beta}\, x^\alpha x^\beta = -\varkappa_{\mathrm{dS}}^{-2}\}, \quad \alpha, \beta = 0, 1, 2, 3, 4,$$

where $\eta_{\alpha\beta}$ =diag$(1, -1, -1, -1, -1)$

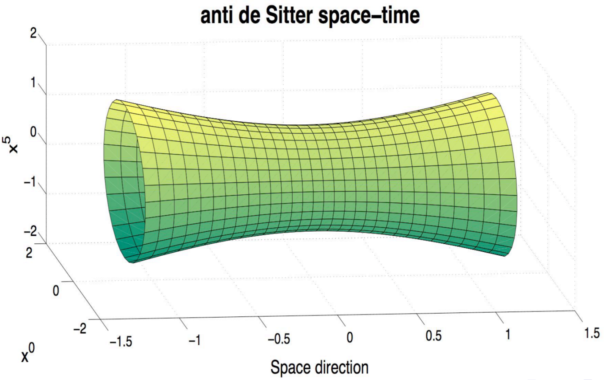


Anti de Sitter geometry

- Anti de Sitter space can be viewed as a one-sheeted hyperboloid embedded in another five-dimensional space with different metric:

$$M_{AdS} \equiv \{x \in \mathbb{R}^5;\; x^2 = \eta_{\alpha\beta}\, x^\alpha x^\beta = \varkappa_{\mathrm{AdS}}^{-2}\}\,, \quad \alpha, \beta = 0, 1, 2, 3, 5,$$

where $\eta_{\alpha\beta}$ =diag$(1, -1, -1, -1, 1)$.



- The de Sitter relativity group is $G = \mathrm{SO}_0(1,4)$ or its universal covering $\mathrm{Sp}(2,2)$

$$\mathrm{Sp}(2,2) = \left\{ g = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \;;\; a,b,c,d \in \mathbb{H},\; \det(\underline{g}) = 1,\; g^{\dagger}\gamma^0 g = \gamma^0 \right\},$$

$g^{\dagger} = \tilde{g}^{t}$ where $\tilde{g}$ is the *quaternionic conjugate* of g and $\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

- From $g^{\dagger}\gamma^0 g = \gamma^0$, one derives

$$|a|^2 - |c|^2 = 1\,, \qquad |d|^2 - |b|^2 = 1\,, \qquad \tilde{a}b = \tilde{c}d\,,$$

- Equivalently

$$|a|^2 - |b|^2 = 1\,, \qquad |d|^2 - |c|^2 = 1\,, \qquad a\tilde{c} = b\tilde{d}\,.$$

which implies $|a| = |d|\,, \qquad |b| = |c|$

Homomorphism between $SO_0(1,4)$ and $Sp(2,2)$

- To $x \in \mathbb{R}^5$ associate the matrix $\not{x}$ built from the Clifford algebra $\{\gamma^\alpha\}$ determined by: $\gamma^\alpha\gamma^\beta + \gamma^\beta\gamma^\alpha = 2\eta^{\alpha\beta}\mathbb{1}_4$

$$\not{x} = x^\alpha\gamma_\alpha = \begin{pmatrix} x^0 & -x \\ \tilde{x} & -x^0 \end{pmatrix}, \qquad \not{x} = \gamma_0\not{x}^\dagger\gamma_0\,, \quad \not{x}^2 = \left((x^0)^2 - |\mathbf{x}|^2\right)\mathbb{1}_4 = -(x)^2\mathbb{1}_4$$

with $x = (x^4, \vec{x}) \in \mathbb{H}$

- $\not{x}$ uniquely determines a point $x \in \mathbb{R}^5$ such that :

$$x^\alpha = \frac{1}{4}\mathrm{tr}\left(\gamma^\alpha\not{x}\right).$$

- $Sp(2,2)$ acts on $\mathbb{R}^5$ as

$$\not{x}' = g\not{x}g^{-1} = \begin{pmatrix} x'^0 & -x' \\ \widetilde{x'} & -x'^0 \end{pmatrix}.$$

and so

$$x'^\alpha = \frac{1}{4}\mathrm{tr}\left(\gamma^\alpha\not{x}'\right) = \frac{1}{4}\mathrm{tr}\left(\gamma^\alpha g\not{x}g^{-1}\right) = \frac{1}{4}\mathrm{tr}\left(\gamma^\alpha g\gamma_\beta g^{-1}\right)x^\beta\,. \tag{1}$$

- Hence, $Sp(2,2)$ is two-to-one homomorphic to $SO_0(1,4)$, with the kernel isomorphic to $\mathbb{Z}^2$:

$$Sp(2,2)/\mathbb{Z}^2 \sim SO_0(1,4)\,.$$

de Sitter geometry

- A familiar realization of the Lie algebra is that one generated by the ten Killing (pseudo-rotation generators) vector fields

$$K_{\alpha\beta} = x_\alpha \partial_\beta - x_\beta \partial_\alpha .$$

- BUT there is no globally time-like Killing vector in de Sitter, the adjective time-like (resp. space-like) referring to the Lorentzian four-dimensional metric induced by that of the bulk.
- In a unitary representation the ten Killing vectors are represented as (essentially) self-adjoint operators in Hilbert space of (spinor-)tensor valued functions on dS, square integrable with respect to some invariant K-G like inner product :

$$K_{\alpha\beta} \longrightarrow L_{\alpha\beta} = M_{\alpha\beta} + S_{\alpha\beta} ,$$

where the orbital part is $M_{\alpha\beta} = -\mathrm{i}(x_\alpha \partial_\beta - x_\beta \partial_\alpha)$ and the spinorial part $S_{\alpha\beta}$ acts on the indices of functions in a certain permutational way.

dS Unitary Irreducible Representations (UIR)

Like for the UIR's of the Poincaré group, there are two Casimir operators, the eigenvalues[1] of which determine completely the UIR's :

$$Q^{(1)}_{\mathrm{dS}} = -\frac{1}{2} L_{\alpha\beta} L^{\alpha\beta},$$

with eigenvalues

$$\langle Q^{(1)}_{\mathrm{dS}} \rangle = -(q+1)(q-2) - p(p+1)\,.$$

and

$$Q^{(2)}_{\mathrm{dS}} = -W_\alpha W^\alpha,\ W_\alpha = -\frac{1}{8}\epsilon_{\alpha\beta\gamma\delta\eta} L^{\beta\gamma} L^{\delta\eta},$$

with eigenvalues

$$\langle Q^{(2)}_{\mathrm{dS}} \rangle = -q(q-1)p(p+1)\,.$$

[1] In the Thomas-Newton-Dixmier notations, J. Dixmier, *Bull. Soc. Math. France*, **89**, 9 (1961).

dS UIR classification (Dixmier, Takahashi)

- **The discrete series** $\Pi^{\pm}_{p,q}$,
  defined by p and q having integer or half-integer values, $p \geq q$
  (i) *The scalar case* $\Pi_{p,0}$, $p = 1, 2, \cdots$;
  (ii) *The spinorial case* $\Pi^{\pm}_{p,q}$, $q > 0$, $p = \frac{1}{2}, 1, \frac{3}{2}, 2, \cdots, q = p, p-1, \cdots, 1$ or $\frac{1}{2}$

- **The principal and complementary series** $U_{\mathrm{dS}}(\varsigma_{\mathrm{dS}}, s)$,
  where $p = s$ has a spin meaning.
  We put $\varsigma^2_{\mathrm{dS}} + \frac{1}{4} = q(1-q)$, i.e. $q = \frac{1}{2} \pm \mathrm{i}\varsigma_{\mathrm{dS}}$
  (i) *The scalar case* for which
    (a) $-9/4 < \varsigma^2_{\mathrm{dS}} < 0$ for the complementary series;
    (b) $0 \leq \varsigma^2_{\mathrm{dS}}$ for the principal series.
  (ii) *The spinorial case* for which
    (a) $-1/4 < \varsigma^2_{\mathrm{dS}} < 0$, $s = 1, 2, \cdots$, for the complementary series,
    (b) $0 \leq \varsigma^2_{\mathrm{dS}}$, $s = 1, 2, \cdots$, for the integer spin principal series,
    (c) $0 < \varsigma^2_{\mathrm{dS}}$, $s = \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \cdots$, for the half-integer spin principal series.

Anti de Sitter space-time and complex quaternions

- In order to display the homomorphism between $\mathrm{SO}_0(2,3)$ and its two-fold covering $\mathrm{Sp}(4,\mathbb{R})$, namely the real symplectic group, one associates the following 4×4 complex matrix with any 5-uple y^α in $\mathbb{R}^{2+3}$,

$$y = (y^\alpha) \mapsto \Gamma(y) = \begin{pmatrix} y_+ \mathbb{1}_2 & \boldsymbol{Y} \\ -\boldsymbol{Y} & y_- \mathbb{1}_2 \end{pmatrix}$$

with $y_\pm = y^5 \pm \mathrm{i}y^0$ and $\boldsymbol{Y} = \begin{pmatrix} \mathrm{i}y^3 & \mathrm{i}y^1 - y^2 \\ \mathrm{i}y^1 + y^2 & -\mathrm{i}y^3 \end{pmatrix}$

- We see that $\boldsymbol{Y}$ is element of the complex quaternionic algebra $\mathbb{H}_\mathbb{C} \sim \mathbb{H}\otimes\mathbb{C} \sim \mathcal{M}_2(\mathbb{C})$:

$$\mathbb{H}_\mathbb{C} \ni z = (z^4, \boldsymbol{z}) \mapsto Z(z) = \begin{pmatrix} z^4 + \mathrm{i}z^3 & \mathrm{i}z^1 - z^2 \\ \mathrm{i}z^1 + z^2 & z^4 - \mathrm{i}z^3 \end{pmatrix} \equiv Z$$

with $\det Z = \det z$, and

$$\bar{z} = \left(\overline{z^4}, \bar{\boldsymbol{z}}\right)\ , \quad \tilde{z} = (z^4, -\boldsymbol{z})\,, \quad z^* = \bar{\tilde{z}} = \tilde{\bar{z}}\,,$$

The AdS group i $G = \mathrm{SO}_0(2,3)$ or its two-fold covering $\mathrm{Sp}(4,\mathbb{R})$ with complex quaternions and its action on AdS

- The elements of $\mathrm{Sp}(4,\mathbb{R})$ are 2×2 complex quaternionic matrices

$$\mathrm{Sp}(4,\mathbb{R}) \ni g = \begin{pmatrix} a & b \\ -\bar{b} & \bar{a} \end{pmatrix}, \quad a, b \in \mathbb{H}_{\mathbb{C}},$$

  such that the inverse g^{-1} is given by

$$g^{-1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} {}^{t}\tilde{g} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} a^* & \tilde{b} \\ -b^* & \tilde{a} \end{pmatrix}.$$

- The complex quaternionic entries of $g = \begin{pmatrix} a & b \\ -\bar{b} & \bar{a} \end{pmatrix} \in \mathrm{Sp}(4,\mathbb{R})$ obey

$$aa^* - bb^* = 1 \quad \text{and} \quad a\tilde{b} = -b\tilde{a}$$

  (or equivalently $a^*a - \tilde{b}\bar{b} = 1$ and $a^*b = -\tilde{b}\bar{a}$), which entails that $a\tilde{b}$ and a^*b are pure complex quaternions.

- Action of $\mathrm{Sp}(4,\mathbb{R})$ on AdS is given by

$$\mathrm{Sp}(4,\mathbb{R}) \ni g \,:\, \Gamma(y) \mapsto \Gamma(y') = g\,\Gamma(y)\,{}^{t}\tilde{g}\,.$$

- There results the homomorphism $g \in \mathrm{Sp}(4,\mathbb{R}) \mapsto R_g \in \mathrm{SO}_0(2,3)$:

$$\Gamma(y') = g\,\Gamma(y)\,{}^{t}\tilde{g} = \Gamma(R_g y)\,.$$

AdS Lie algebra

- Like for dS, a realization of the Lie algebra is that one generated by the ten Killing vectors
$$K_{\alpha\beta} = x_\alpha \partial_\beta - x_\beta \partial_\alpha .$$

- Contrarily to dS, there is one globally time-like Killing vector in anti de Sitter, namely K_{50}.

- The compact nature of the associated one-parameter group (it is just $\mathrm{SO}(2) \simeq \mathrm{U}(1)$ or its double covering) can raise problems. The latter can be circumvented by dealing with the universal covering $\widetilde{G} = \widetilde{\mathrm{SO}_0(2,3)}$ in which the "time" $\mathrm{SO}(2)$ subgroup becomes $\mathbb{R}$.

AdS UIR's

- Physically meaningful UIR's of the de Anti de Sitter group are found in the (holomorphic) discrete series and in its lower limits
- Like in dS, the infinitesimal generators read as:

$$K_{\alpha\beta} \longrightarrow L_{\alpha\beta} = M_{\alpha\beta} + S_{\alpha\beta},$$

where the orbital part is $M_{\alpha\beta} = -\mathrm{i}(x_\alpha\partial_\beta - x_\beta\partial_\alpha)$ and the spinorial part $S_{\alpha\beta}$ acts on the indices of functions in a certain permutational way

- In the case of the discrete series and its lower limit, these UIR's are denoted $U_{\mathrm{AdS}}(\varsigma_{\mathrm{AdS}}, s)$ with $2s \in \mathbb{N}$ and $\varsigma_{\mathrm{AdS}} \geq s+1$ (at the exception of a few cases). The label s is for spin and $\dfrac{\hbar\varkappa}{c}\varsigma_{\mathrm{AdS}}$ for rest "energy" .

For UIR in the strictu senso discrete series of the double covering $\mathrm{Sp}(4,\mathbb{R})$, the parameter ς_{AdS} is such that $2\varsigma_{\mathrm{AdS}} \in \mathbb{N}$ whilst for "discrete" series UIR of the universal covering $\widetilde{\mathrm{SO}_0(2,3)}$ this parameter assumes its values in $[s+1,\infty)$.

Classification of AdS UIR in the discrete series

- Eigenvalues of the two Casimir operators determine completely the UIR's :

$$Q^{(1)}_{\mathrm{AdS}} = -\frac{1}{2} L_{\alpha\beta} L^{\alpha\beta} ,$$

with eigenvalues

$$\langle Q^{(1)}_{\mathrm{AdS}} \rangle = \varsigma_{\mathrm{AdS}}(\varsigma_{\mathrm{AdS}} - 3) + s(s+1) \, .$$

and

$$Q^{(2)}_{\mathrm{AdS}} = -W_\alpha W^\alpha , \ W_\alpha = -\frac{1}{8}\epsilon_{\alpha\beta\gamma\delta\eta} L^{\beta\gamma} L^{\delta\eta} ,$$

with eigenvalues

$$\langle Q^{(2)}_{\mathrm{AdS}} \rangle = -(\varsigma_{\mathrm{AdS}} - 1)(\varsigma_{\mathrm{AdS}} - 2) s(s+1) \, .$$

- Among the AdS UIR $U_{\mathrm{AdS}}(\varsigma_{\mathrm{AdS}}, s)$, one must distinguish between those for which $\varsigma_{\mathrm{AdS}} > s+1$, and the following important limit cases [2]
  - (i) *The limit scalar cases* $U_{\mathrm{AdS}}(1,0)$ and $U_{\mathrm{AdS}}(\frac{1}{2},0)$. The latter is called the "Rac"
  - (ii) *The limit spinorial or tensorial cases* $U_{\mathrm{AdS}}(s+1,s)$ and $U_{\mathrm{AdS}}(1,\frac{1}{2})$. The latter is called the "Di"

[2] M. Flato and C. Fronsdal, *Phys. Lett. B* **97**

Minkowskian content of dS and AdS elementary systems

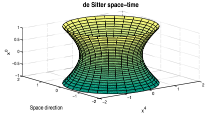
- Now, we wish to go further into the interpretative question of mass and “energy at rest” in a dS/AdS background
- The crucial question to be addressed concerns the interpretation of the dS/AdS UIR's (or quantum AdS and dS elementary systems) from a (asymptotically) Minkowskian point of view
- We mean by this the study of the contraction limit $\varkappa \to 0$ or equivalently $\Lambda \to 0$ of these representations, which is the quantum counterpart of the following geometrical and group contractions

“A physical theory that treats spacetime as Minkowskian flat must be obtainable as a well-defined limit of a more general physical theory, for which the assumption of flatness is not essential.” Fronsdal (1965)

Flat limit of de Sitter geometry (optional)

- Example of global coordinates on dS, $\tau := ct \in \mathbb{R}, \vec{n} \in \mathbb{S}^2, \alpha := \varkappa_{\mathrm{dS}} r \in [0, \pi]$:

$$M_{dS} \ni x := (x^0, \vec{x} = (x^1, x^2, x^3), x^4) \\ = (\varkappa_{\mathrm{dS}}^{-1} \sinh(\varkappa_{\mathrm{dS}} ct),\ \varkappa_{\mathrm{dS}}^{-1} \cosh(\varkappa_{\mathrm{dS}} ct) \sin(\varkappa_{\mathrm{dS}} r)\vec{n}, \\ \varkappa_{\mathrm{dS}}^{-1} \cosh(\varkappa_{\mathrm{dS}} ct) \cos(\varkappa_{\mathrm{dS}} r)).$$



- $\lim_{\varkappa_{\mathrm{dS}} \to 0} M_{dS} = M_0$, the Minkowski spacetime tangent to M_{dS} at, say, the de Sitter origin point $O_{dS} = (0, \vec{0}, \varkappa_{\mathrm{dS}}^{-1})$, since then

$$M_{dS} \ni x \underset{\varkappa_{\mathrm{dS}} \to 0}{\approx} (t, \vec{r} = r\,\vec{n}, \varkappa_{\mathrm{dS}}^{-1})$$

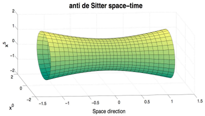
- $\lim_{\varkappa_{\mathrm{dS}} \to 0} \mathrm{Sp}(2,2) = \mathcal{P}_+^{\uparrow}(1,3) = M_0 \rtimes \mathrm{SL}(2,\mathbb{C})$, the Poincaré group.

- The ten de Sitter Killing vectors (in the Wigner-Inonü sense) contract to their Poincaré counterparts $K_{\mu\nu}, \Pi_\mu, \mu = 0, 1, 2, 3$, after rescaling the four $K_{4\mu} \longrightarrow \Pi_\mu = \varkappa_{\mathrm{dS}} K_{4\mu}$.

The same for the flat limit of Anti de Sitter geometry (optional)

- Example of global coordinates in AdS : $\tau := \varkappa_{\rm AdS} ct \in [0, 2\pi), r \in [0, \infty), \vec{n} \in \mathbb{S}^2$,

$$
\begin{aligned}
M_{AdS} \ni x :=&(x^0, \vec{x} = (x^1,\, x^2, x^3),\, x^5) \\
=&(\varkappa_{\rm AdS}^{-1} \cosh\,(\varkappa_{\rm AdS} r)\ \sin(\varkappa_{\rm AdS} ct), \varkappa_{\rm AdS}^{-1} \sinh(\varkappa_{\rm AdS} r)\, \vec{n}, \\
& x^5 = \varkappa_{\rm AdS}^{-1} \cosh(\varkappa_{\rm AdS} r)\ \cos(\varkappa_{\rm AdS} ct).
\end{aligned}
$$



- $\lim_{\varkappa_{\rm AdS}\to 0} M_{AdS} = M_0$, the Minkowski spacetime tangent to M_{AdS} at, say, the de Sitter origin point $O_{AdS} = (0, \vec{0}, \varkappa_{\rm AdS}^{-1})$, since then

$$
M_{AdS} \ni x \underset{\varkappa_{\rm AdS}\to 0}{\approx} (t = \tau, \vec{r} = r\vec{n}, \varkappa_{\rm AdS}^{-1})
$$

- $\lim_{\varkappa_{\rm AdS}\to 0} \mathrm{Sp}(4, \mathbb{R}) = \mathcal{P}_+^\uparrow(1, 3) = M_0 \rtimes \mathrm{SL}(2, \mathbb{C})$.

- The ten de Sitter Killing vectors contract to their Poincaré counterparts $K_{\mu\nu}$, Π_μ, $\mu = 0, 1, 2, 3$, after rescaling the four $K_{5\mu} \longrightarrow \Pi_\mu = \varkappa_{\rm AdS} K_{5\mu}$.

Mass in Minkowski: a non ambiguous notion (in the Wigner sense)

- $\mathcal{P}^{\gtrless}(m,s)$ stand for the positive (resp. negative) energy Wigner UIR's of the Poincaré group with mass m ($\equiv$ proper mass $\equiv$ energy at rest, $c=1$) and spin s : no ambiguity about the mass m!
- $\mathcal{P}^{\gtrless}(0,s)$ stand for the Poincaré massless cases where s reads for helicity.
- In this massless case, conformal invariance leads us to deal with the discrete series representations (and their lower limits) of the (universal covering of the) conformal group or its double covering $\mathrm{SO}_0(2,4)$ or its fourth covering $\mathrm{SU}(2,2)$.
- These (conformal meaningful) UIR's are denoted by $\mathcal{C}^{\gtrless}(\varsigma, j_1, j_2)$, where $(j_1, j_2) \in \mathbb{N}/2 \times \mathbb{N}/2$ labels the UIR's of the $\mathrm{SU}(2) \times \mathrm{SU}(2)$ subgroup and ς stems for the positive (resp. negative) conformal energy.

The notion of mass in de Sitter is ambiguous!

- The notion of mass in “desitterian Physics” may appear confusing. Nevertheless a mass formula has been proposed by Garidi[3] in terms of the dS RUI parameters p and q:

$$\boxed{m_{\mathrm{dS}}^2 = \frac{\hbar^2 \varkappa_{\mathrm{dS}}^2}{c^2}\left(\langle Q_{\mathrm{dS}}^{(1)}\rangle - \langle Q_{\mathrm{dS}}^{(1)}|_{p=q}\rangle\right) = \frac{\hbar^2 \varkappa_{\mathrm{dS}}^2}{c^2}\left(\varsigma_{\mathrm{dS}}^2 + \left(s-\frac{1}{2}\right)^2\right)}\,.$$

- The minimal value assumed by the eigenvalues of the first Casimir in the set of RUI in the discrete series is precisely reached at $s = 1/2 + \mathrm{i}\varsigma_{\mathrm{dS}}$, which corresponds to the “conformal” massless case

- Controlling the validity of such a formula from a Minkowskian observer amounts to understand the contraction (mathematically non trivial en terms of sequences of Hilbert spaces)

$$\mathrm{dS\ UIR} \longrightarrow \text{Poincaré UIR}$$

[3] T. Garidi, *What is mass in desitterian Physics?* hep-th/0309104

de Sitter contraction limits. Massive case

- Solely the principal series representations are involved here (from where the name of de Sitter "massive representations")
- In terms of the representation parameter ς_{dS} and for a spin s, the Casimir eigenvalue and Garidi mass read respectively:

$$\langle Q^{(1)}_{\mathrm{dS}}\rangle = -s(s+1) + \varsigma^2_{\mathrm{dS}} + \frac{9}{4},$$

$$m_{\mathrm{dS}} = \frac{\hbar\varkappa_{\mathrm{dS}}}{c}\left(\varsigma^2_{\mathrm{dS}} + \left(s-\frac{1}{2}\right)^2\right)^{1/2}$$

- Then the contraction dS $\to$ Poincaré in terms of masses has to be understood as

$$\varkappa_{\mathrm{dS}} \to 0 \quad \varsigma_{\mathrm{dS}} \to \infty\,, \quad \text{while fixing} \quad \varsigma_{\mathrm{dS}}\hbar\varkappa_{\mathrm{dS}}/c = m_{\mathrm{Poincaré}} \equiv m\,.$$

de Sitter contraction limits. Massive case continued (optional)

- Actually we have the following general result on contraction of dS principal series representations[4]:

$$U_{\mathrm{dS}}(\varsigma_{\mathrm{dS}}, s) \underset{\substack{\varkappa_{\mathrm{dS}}\to 0, |\varsigma_{\mathrm{dS}}|\to\infty \\ |\varsigma_{\mathrm{dS}}|\varkappa_{\mathrm{dS}} = \frac{mc}{\hbar}}}{\longrightarrow} c_{>}\mathcal{P}^{>}(m,s) \oplus c_{<}\mathcal{P}^{<}(m,s),$$

- One of the "coefficients" among $c_{<}$, $c_{>}$ can be fixed to 1 whilst the other one will vanish. Note that $m = m_{\mathrm{dS}} + O(\varkappa_{\mathrm{dS}})$. (a choice left to a "local tangent" observer).
- Note also here the evidence of the energy ambiguity in de Sitter relativity, exemplified by the possible breaking of dS irreducibility into a direct sum of two Poincaré UIR's with positive and negative energy respectively
- This phenomenon is linked to the existence in the de Sitter group of a specific discrete symmetry which sends any point $(x^0, \mathrm{P}) \in M_{dS}$ into its mirror image $(x^0, -\mathrm{P}) \in M_{dS}$ with respect to the x^0-axis.
- Under such a symmetry the four generators L_{a0}, $a = 1, 2, 3, 4$, (and particularly L_{40} which contracts to energy operator!) transform into their respective opposite $-L_{a0}$, whereas the six L_{ab}'s remain unchanged.

[4] J. Mickelsson and J. Niederle, *Commun. Math. Phys.* **27** (1972)
T. Garidi, E. Huguet, and J. Renaud, *Phys. Rev. D* **67** (2003), gr-qc/0304031

de Sitter contraction limits. Massless (conformal) case (optional)

From Barut A. O., Böhm A., *J. Math. Phys.*, **11** (1970) 2938 *Reduction of a class of O$(4,2)$ representations with respect to SO$(4,1)$ and SO$(3,2)$*

Here we have $m_{\mathrm{dS}} = 0$ for all involved representations. Now, we must distinguish between

(i) the scalar massless case involves the unique complementary series UIR $U_{\mathrm{dS}}(\pm \mathrm{i}/2, 0)$ (for which $\langle Q^{(1)}\rangle_{\mathrm{dS}} = 2$) to be contractively Poincaré significant

$$U_{\mathrm{dS}}(\pm \mathrm{i}/2, 0) \hookrightarrow \begin{array}{c} \mathcal{C}^{>}(1,0,0) \\ \oplus \\ \mathcal{C}^{<}(-1,0,0) \end{array} \xrightarrow{\varkappa=0} \begin{array}{c} \mathcal{C}^{>}(1,0,0) \\ \oplus \\ \mathcal{C}^{<}(-1,0,0) \end{array} \begin{array}{c} \hookleftarrow \\ \\ \hookleftarrow \end{array} \begin{array}{c} \mathcal{P}^{>}(0,0) \\ \oplus \\ \mathcal{P}^{<}(0,0), \end{array}$$

(ii) the spinorial case where are involved all representations $\Pi^{\pm}_{s,s}$, $s > 0$ for which $\langle Q^{(1)}\rangle_{\mathrm{dS}} = -2(s^2 - 1)$ and lying at the lower limit of the discrete series (the arrows $\hookrightarrow$ designate unique extension.)

$$\Pi^{+}_{s,s} \hookrightarrow \begin{array}{c} \mathcal{C}^{>}(s+1,s,0) \\ \oplus \\ \mathcal{C}^{<}(-s-1,s,0) \end{array} \xrightarrow{\varkappa=0} \begin{array}{c} \mathcal{C}^{>}(s+1,s,0) \\ \oplus \\ \mathcal{C}^{<}(-s-1,s,0) \end{array} \begin{array}{c} \hookleftarrow \\ \\ \hookleftarrow \end{array} \begin{array}{c} \mathcal{P}^{>}(0,s) \\ \oplus \\ \mathcal{P}^{<}(0,s), \end{array}$$

$$\Pi^{-}_{s,s} \hookrightarrow \begin{array}{c} \mathcal{C}^{>}(s+1,0,s) \\ \oplus \\ \mathcal{C}^{<}(-s-1,0,s) \end{array} \xrightarrow{\varkappa=0} \begin{array}{c} \mathcal{C}^{>}(s+1,0,s) \\ \oplus \\ \mathcal{C}^{<}(-s-1,0,s) \end{array} \begin{array}{c} \hookleftarrow \\ \\ \hookleftarrow \end{array} \begin{array}{c} \mathcal{P}^{>}(0,-s) \\ \oplus \\ \mathcal{P}^{<}(0,-s), \end{array}$$

An exceptional case: the dS "massless" minimally coupled field (optional)

- All other representations have either non-physical Poincaré contraction limit or have no contraction limit at all
- In particular, we have for the so-called *massless minimally coupled field*[5] which corresponds to the UIR $\Pi^+_{1,0}$ lying at the lowest limit of the discrete series the following values for Casimir eigenvalue and Garidi mass:

$$\langle Q^{(1)}_{\mathrm{dS}} \rangle = 0, \; m_{\mathrm{dS}} = 0.$$

- This representation, and hence the corresponding field, is exceptional under many aspects:
  - (i) it is the only one among all non massless dS representations for which the Garidi mass vanishes
  - (ii) it is part of an indecomposable structure issued from the existence of (constant) gauge solutions
  - (iii) it has been thought to play a role in inflation theories
  - (iv) it is part of the Gupta-Bleuler structure for the massless spin 1 dS field (de Sitter QED) described by the UIR's $\Pi^+_{1,1}$
  - (v) It is the elementary brick for the construction of the massless spin 2 dS fields (de Sitter linear gravity) described by the UIR's $\Pi^+_{2,2}$
  - (vi) the corresponding covariant quantum field theory requires a specific treatment due precisely to its indecomposable nature[6]

[5] The term "massless" refers to the description of a scalar field ϕ with "mass m" and coupling ξ to the curvature scalar R by the action $S = \frac{1}{2}\int d^4x\,\sqrt{-g}\,[\nabla_a\phi\,\nabla^a\phi - m^2\,\phi^2 - \xi\,R\,\phi^2]$.

[6] J. P. G., J. Renaud and M. V. Takook, *Class. Quantum. Grav.*, **17** (2000) 1415.

Contraction limits Anti de Sitter → Minkowski. The massive case

- A "mass" formula [7] analogous to the Garidi one can be proposed in the case of the AdS discrete series. It precisely vanishes for massless AdS fields:

$$m^2_{\mathrm{AdS}} = \frac{\hbar^2 \varkappa^2_{\mathrm{AdS}}}{c^2}\left(\langle Q^{(1)}_{\mathrm{AdS}}\rangle - \langle Q^{(1)}_{\mathrm{AdS}}|_{\varsigma_{\mathrm{AdS}}=s+1}\rangle\right) = \frac{\hbar^2 \varkappa^2_{\mathrm{AdS}}}{c^2}\left[\left(\varsigma_{\mathrm{AdS}} - \frac{3}{2}\right)^2 - \left(s - \frac{1}{2}\right)^2\right]$$

- Solely the (holomorphic) discrete series representations $U_{\mathrm{AdS}}(\varsigma, s)$ with $\varsigma_{\mathrm{AdS}} > s+1$ are involved here.

$$U_{\mathrm{AdS}}(\varsigma_{\mathrm{AdS}}, s) \underset{\substack{\varkappa_{\mathrm{AdS}}\to 0,\, \varsigma_{\mathrm{AdS}}\to\infty \\ \varsigma_{\mathrm{AdS}}\varkappa_{\mathrm{AdS}} = \frac{mc}{\hbar}}}{\longrightarrow} \mathcal{P}^{>}(m, s)$$

- Note here that there is no energy ambiguity in Anti de Sitter relativity (there are other ambiguities!). If we wished to get the negative energy Poincaré representations, we would instead have chosen the representations in the *antiholomorphic* discrete series (in which the spectrum of the compact generator L_{50} is bounded above by $-\varsigma_{\mathrm{AdS}}$, $\varsigma_{\mathrm{AdS}} > 0$)

[7] J.-P. G. and M. Novello, *J. Phys. A: Math. Theor.* **41** (2008); The Nature of Λ and the Mass of the Graviton: A Critical View, J.-P. G. and M. Novello *Int. J. Mod. Phys. A* **26**, (2011)

Contraction limits anti de Sitter → Minkowski. The massless case (optional)

We must distinguish between

(i) the scalar massless case which involves the UIR $U_{\text{AdS}}(1,0)$

$$U_{\text{AdS}}(1,0) \quad \hookrightarrow \quad \mathcal{C}^{>}(1,0,0) \quad \overset{\varkappa=0}{\longrightarrow} \quad \mathcal{C}^{>}(1,0,0) \quad \hookleftarrow \quad \mathcal{P}^{>}(0,0).$$

(ii) the spinorial-tensorial massless case in which are involved all representations $U_{\text{AdS}}(s+1,s),\ s>0$ lying at the lower limit of the holomorphic discrete series.

$$U_{\text{AdS}}(s+1,s) \quad \hookrightarrow \quad \begin{matrix} \mathcal{C}^{>}(s+1,s,0) \\ \oplus \\ \mathcal{C}^{>}(s+1,0,s) \end{matrix} \quad \overset{\varkappa=0}{\longrightarrow} \quad \begin{matrix} \mathcal{C}^{>}(s+1,s,0) & \hookleftarrow & \mathcal{P}^{>}(0,s) \\ \oplus & & \oplus \\ \mathcal{C}^{>}(s+1,0,s) & \hookleftarrow & \mathcal{P}^{>}(0,-s). \end{matrix}$$

Here, there is no ambiguity concerning energy, but there is ambiguity concerning helicity, since the latter is not defined in AdS.

Two exceptional case: the AdS "singletons" (optional)

- All other representations have either non-physical Poincaré contraction limit or have no contraction limit at all
- It should be also noted that, like for de Sitter, there exists a unique UIR, among all non massless AdS representations, for which m_{AdS} vanishes, namely the UIR $D(2,0)$ in the discrete series.
- In particular, we have for the *Rac* and *Di* fields the following respective values for Casimir eigenvalue and Garidi mass:

$$\langle Q^{(1)}_{\mathrm{AdS}}\rangle = -\frac{5}{4}, \quad m_{\mathrm{AdS}} = \frac{\sqrt{3}}{2}\frac{\hbar\varkappa_{\mathrm{AdS}}}{c}, \text{ (Rac)},$$

$$\langle Q^{(1)}_{\mathrm{AdS}}\rangle = -\frac{5}{4}, \quad m_{\mathrm{AdS}} = \frac{\hbar\varkappa_{\mathrm{AdS}}}{2c}, \text{ (Di)}.$$

- These representations, and hence their corresponding fields, are also exceptional under many aspects, particularly due to the fact that AdS massless fields are composite in terms of these singletons[8]

[8] M. Flato and C. Fronsdal, *Lett. Math. Phys.* **2** (1978) 421-426

Inertial versus gravitational mass

Now, the contraction Formulae dS & AdS $\to$ Poincaré give us the freedom to write

$$m_{\mathrm{dS}} = m = m_{\mathrm{AdS}}$$

which agrees with the Einstein position that the proper mass of an elementary system should be independent of the geometry of space-time, or equivalently there should not exist any difference between inertial and gravitational mass.

Energy at rest of a free particle in AdS versus dS and Poincaré

- Each **Anti-deSitterian quantum elementary system** (in the Wigner sense) has a rest energy

$$E^{\text{rest}}_{\text{AdS}} = \left[m^2c^4 + \hbar^2 c^2 \varkappa^2_{\text{AdS}} \left(s - \frac{1}{2}\right)^2\right]^{1/2} + \frac{3}{2}\hbar\varkappa_{\text{AdS}}\, c\,, \tag{2}$$

- Hence, to the order of $\hbar$, an AdS elementary system is a deformation of both a relativistic free particle with rest energy mc^2 and a 3d isotropic quantum harmonic oscillator with ground state energy $\frac{3}{2}\hbar\omega_{\text{AdS}}$, with $\omega_{\text{AdS}} := \varkappa_{\text{AdS}} c = \sqrt{\frac{|\Lambda_{\text{AdS}}|}{3}}\, c.$.

- In contrast to AdS, for Poincaré and DS symmetries the energy spectrum is continuous $\geq mc^2$ (in absolute value):

$$E^{\text{rest}}_{\text{dS}} = \pm\left[m^2c^4 - \hbar^2 c^2 \varkappa^2_{\text{dS}} \left(s - \frac{1}{2}\right)^2\right]^{1/2} . \tag{3}$$

- Noticeable simplification in both AdS and dS for **fermions** $s = 1/2$:

$$\text{for dS :} \quad E^{\text{rest}}_{\text{dS}} = \pm mc^2\,, \tag{4}$$

$$\text{for AdS:} \quad E^{\text{rest}}_{\text{AdS}} = mc^2 + \frac{3}{2}\hbar\omega_{\text{AdS}}\,. \tag{5}$$

The choice $E^{\text{rest}}_{\text{dS}} = mc^2$ should be privileged for obvious reasons.

Energy of a free particle in AdS versus dS and Poincaré (continued)

- In the massless case and spin s, we have

$$\text{for dS}: \quad E^{\text{rest}}_{\text{dS}} = \pm \mathrm{i}\hbar\varkappa_{\text{dS}}\, c \left(s - \frac{1}{2}\right)\,, \tag{6}$$

$$\text{for AdS}: \quad E^{\text{rest}}_{\text{AdS}} = \hbar\varkappa_{\text{AdS}} c(s+1)\,. \tag{7}$$

Therefore, while for dS the energy at rest makes sense only for massless fermionic systems and is just zero, for AdS the energy at rest makes sense for any spin, and in particular for **spin 1 massless bosons** we get

$$\boxed{E^{\text{rest}}_{\text{AdS}} = 2\hbar\omega_{\text{AdS}}\,.} \tag{8}$$

- Save the proper energy $mc^2 \geq 0$ common to dS, AdS, and Poincaré, the energy spectrum of a free particle in AdS is like the spectrum of a 3d isotropic quantum harmonic oscillator whose excited states apart from degeneracy are spaced at equal energy intervals of $\hbar\omega_{\text{AdS}} = \hbar\varkappa_{\text{AdS}}\, c$.

To conclude: about dark matter

Some (observational) facts about dark matter

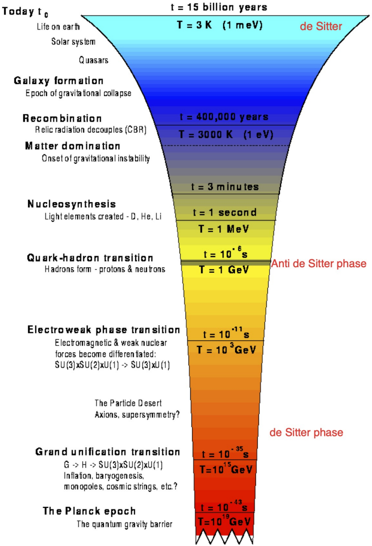
- According to Planck 2015 analysis[9] of CMB power spectrum, our Universe is spatially flat, accelerating, and composed of 5% baryonic matter, 27% cold dark matter (CDM, non baryonic) and 68% dark energy (Λ)[10]
- (Cold) dark matter is observed by its gravitational influence on luminous, baryonic matter
- The dark matter mass halo and the total stellar mass are coupled through a function that varies smoothly with mass (with controversial exception(s) like the recent[11])
- Up to now, all hypothetical particle models (WIMP, Axions, Neutrinos ...) failed direct or indirect detection tests
- Similarly, alternative theories (e.g. MOND) to dark matter have failed to explain clusters and the observed pattern in the CMB.

[9] Planck 2015 results XIII. Cosmological parameters, A& A **594** (2016)

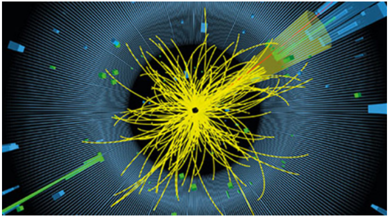
[10] The Search for Dark Matter L. Baudis, *European Review* **26** 70-81 (2017)

[11] A galaxy lacking dark matter van Dokkum et al, Nature Lett. **555** (2018), and references therein

Cosmology chronology : de Sitter and Anti de Sitter phases



Quark-Gluon Plasma: experimental evidence



From Strong interactions News
Protons probe quark-gluon plasma at CMS 13 January 2017

- Theories predicting the existence of quark-gluon plasma were developed in the late 1970s and early 1980s (Satz, Rafelsky, Kapusta, Müller, Letessier...)
- Quark-gluon plasma was detected for the first time at CERN (2000)
- Lead and gold nuclei have been used for collisions yielding QGP at CERN SPS and BNL RHIC, respectively
- The current estimate of the hadronization temperature for light quarks is $T_{cf} = 156.5 \pm 1.5\,\mathrm{MeV} \approx 1.8 \times 10^{12}$ K ("chemical freeze-out temperature").

Cold dark matter: Bose-Einstein condensation of gluons in Anti-de Sitter space time

Gilles Cohen-Tannoudji and J-P Gazeau Universe 2021, 7(11), 402;
https://doi.org/10.3390/universe7110402

- A parallel between dark matter and CMB:
  - CMB $\rightarrow$ photon decoupling, i.e. photons started to travel freely through space rather than constantly being scattered by electrons and protons in plasma (QED effect).
  - Dark matter $\rightarrow$ gluonic component of the quark epoch (quark-gluon plasma) which freely subsists after hadronization within an effective AdS environment (QCD effect)
- As an assembly of N_G non-interacting entities with individual energies $E_n = E^{\text{rest}}_{\text{AdS}} + (n+2)\hbar\omega_{\text{AdS}}$ and degeneracy $g_n = (n+1)(n+3)/2$, those remnant gluons are assumed to form a grand canonical Bose-Einstein ensemble whose the chemical potential μ is, at temperature T, fixed by

$$N_G = \sum_{n=0}^{\infty} \frac{g_n}{\exp\left[\frac{\hbar\omega_{\text{AdS}}}{k_B T}(n+\nu_0-\mu)\right] - 1}\,, \quad \nu_0 := \frac{E^{\text{rest}}_{\text{AdS}}}{\hbar\omega_{\text{AdS}}}\,.$$

- Since this number is very large this gas condensates at temperature

$$T_c \approx \frac{\hbar\omega_{\text{AdS}}}{k_B}\left(\frac{N_G}{\zeta(3)}\right)^{1/3}$$

to become the currently observed dark matter.

Planck 2018 results XIII. Cosmological parameters, *A&A* (2019); arXiv:1807.06209v1 [astro-ph.CO]

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Voici la première image scientifique du télescope James-Webb