Interacting black holes and near horizon symmetries

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Letelier School, December 2021

Black holes

Why study black holes?

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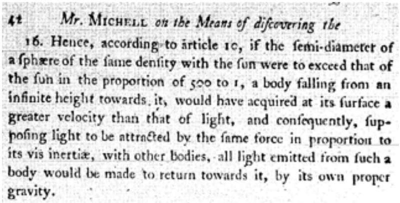
- Physics history

Milestones

1783 First thoughts about black holes (Michell)



42 Mr. MICHELL on the Means of diſcovering the

16. Hence, according to article 10, if the ſemi-diameter of a ſphære of the ſame denſity with the ſun were to exceed that of the ſun in the proportion of 500 to 1, a body falling from an infinite height towards it, would have acquired at its ſurface a greater velocity than that of light, and conſequently, ſuppoſing light to be attracted by the ſame force in proportion to its vis inertiæ, with other bodies, all light emitted from ſuch a body would be made to return towards it, by its own proper gravity.

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Setzt man diese Werte der Funktionen f im Ausdruck (9) des Linienelements ein und kehrt zugleich zu gewöhnlichen Polarkoordinaten zurück, so ergibt sich das Linienelement, welches die strenge Lösung des EINSTEINschen Problems bildet:

$$ds^2 = (1-\alpha/R)dt^2 - \frac{dR^2}{1-\alpha/R} - R^2\left(d\vartheta^2 + \sin^2\vartheta d\phi^2\right), \; R = \left(r^3+\alpha^3\right)^{\frac{1}{3}}. \quad (14)$$

Dasselbe enthält die eine Konstante α, welche von der Größe der im Nullpunkt befindlichen Masse abhängt.

Schwarzschild: Über das Gravitationsfeld eines Massenpunktes

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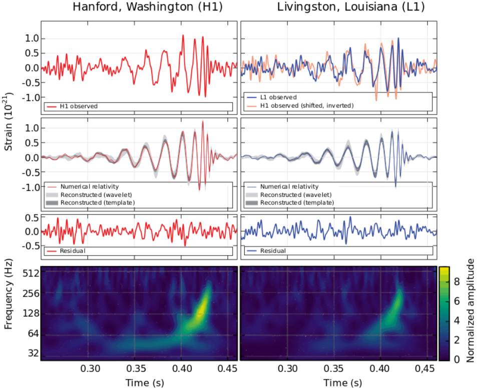

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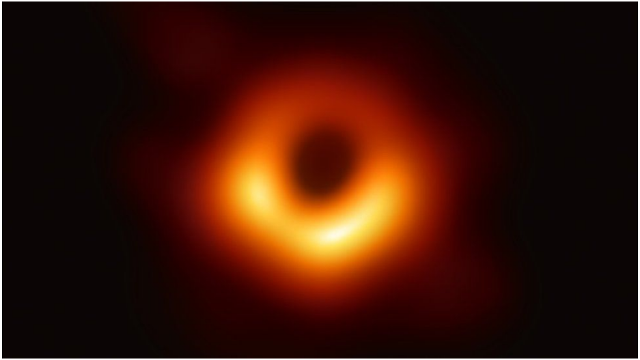
2019 First photo of black hole shadow (EHT)



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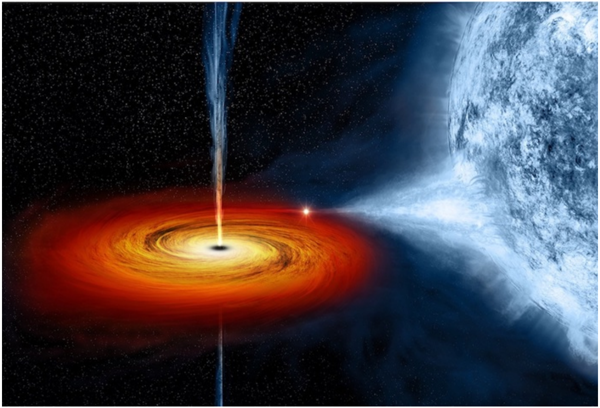

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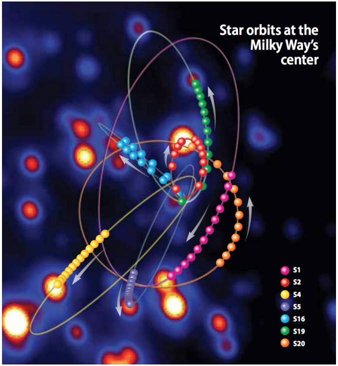

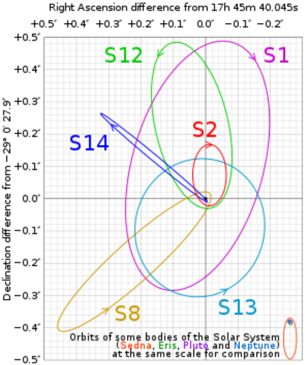

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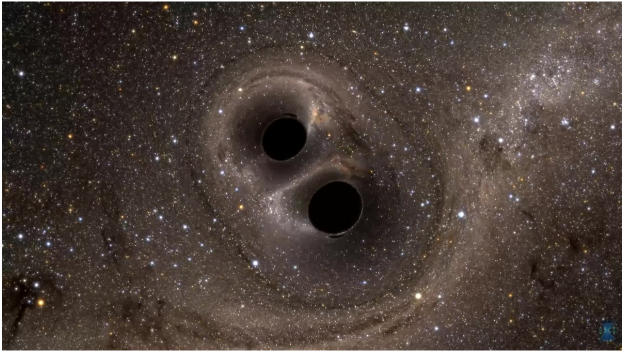

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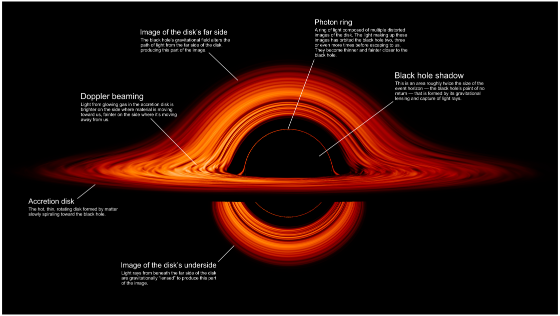

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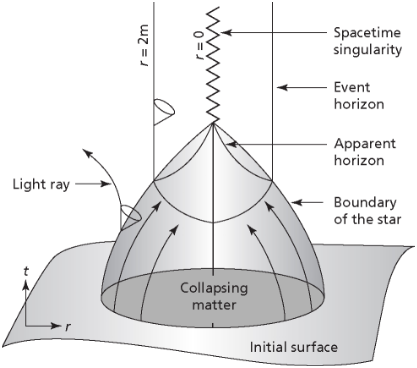

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- Black hole and singularity theorems

Theorem 9.6 (Penrose) *Let (M, g) be a connected globally hyperbolic spacetime with a noncompact Cauchy hypersurface S, satisfying the null energy condition. If S contains a trapped surface Σ then (M, g) is singular.*

Proof Let $t : M \to \mathbb{R}$ be a global time function such that $S = t^{-1}(0)$. The integral curves of grad t, being timelike, intersect S exactly once, and $\partial I^+(\Sigma)$ at most once. This defines a continuous injective map $\pi : \partial I^+(\Sigma) \to S$, whose image is open. Indeed, if $q = \pi(p)$, then all points in some neighborhood of q are images of points in $\partial I^+(\Sigma)$, as otherwise there would be a sequence $q_n \in S$ with $q_n \to q$ such that

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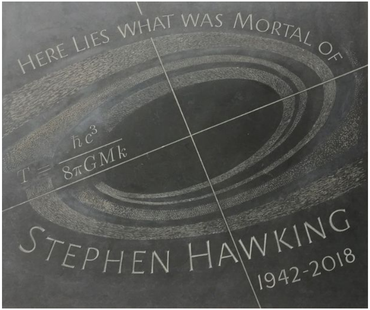

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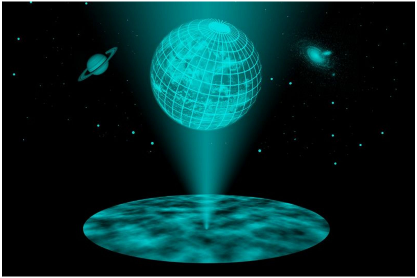
  - Holographic principle and quantum gravity



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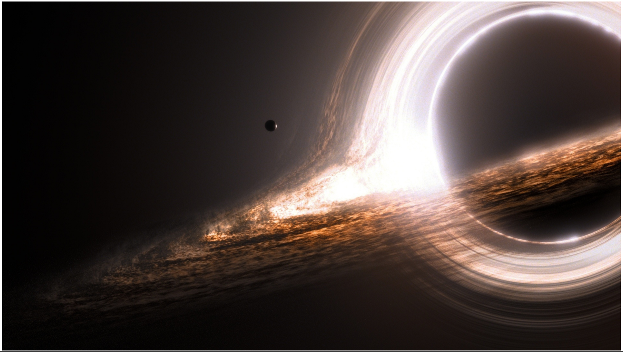
Black holes relevant for astrophysics, cosmology, classical and quantum gravity ... and science fiction



Theoretical idealizations of black holes

Simplest assumption: isolated stationary black hole in vacuum

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- Example: Schwarzschild metric

$$\mathrm{d}s^2 = -\Big(1 - \frac{2M}{r}\Big)\,\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1-\frac{2M}{r}} + r^2\left(\,\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right)$$

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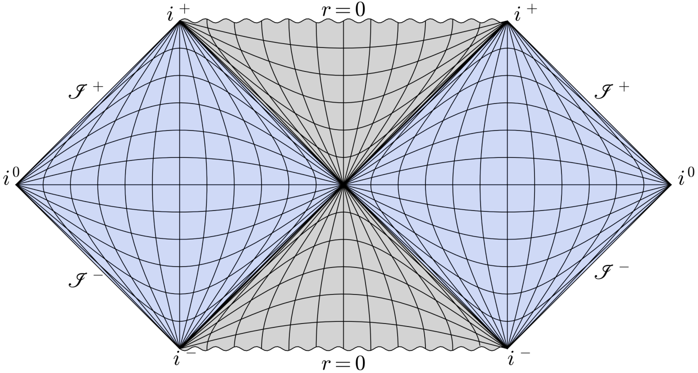

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  - no charge

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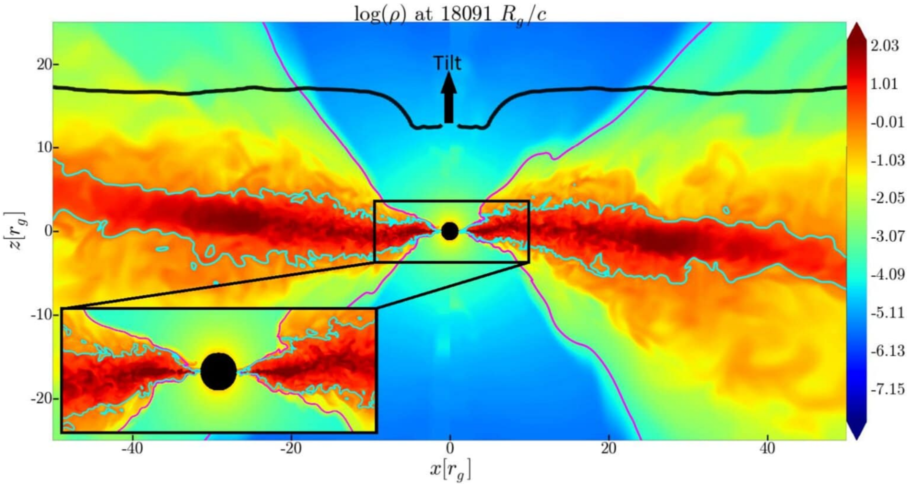
Detecting black holes through accretion disks: matter crucial

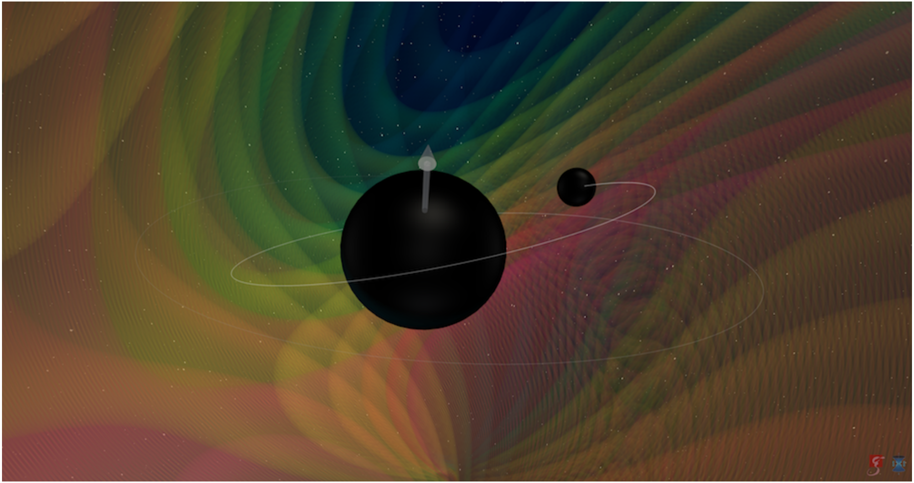
Historically: Shakura, Sunyaev '73. Picture below from 1810.00883



Accretion disk simulations (need general relativity, magnetohydrodynamics, viscosity, plasma physics, ...)

Detecting black holes through gravitational radiation: non-stationary!

Historically: LIGO.



Snapshot of numerical simulation video from LIGO

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- theoretical progress (see next slides)

Lemos–Letelier black hole with disk

δ-like energy momentum tensor in equatorial plane of black hole

Exact general relativistic thin disks around black holes

José P. S. Lemos and Patricio S. Letelier
Phys. Rev. D **49**, 5135 – Published 15 May 1994

ABSTRACT

The formalism for superposing two axially symmetric exact solutions of Einstein field equations, namely, a black hole and a thin disk, is presented. Three different families of disks are analyzed. The most important family gives the first known exact solution for a black hole surrounded by a realistic heavy disk of matter. This family is the last to be analyzed. The matter of the disks is made of counterrotating particles with as many particles rotating to one side as to the other in such a way that the net angular momentum is zero and the disk is static. The first family consists of peculiar disks, in the sense that they are generated by two opposite dipoles. The particles of the disk have no pressure or centrifugal support. However, when there is a central black hole, centrifugal balance in the form of counterrotation appears. The second family is formed by disks of finite extent, the Morgan and Morgan disks. Within this family there are three parameters to play with: the black hole and disk masses, and the disk radius. These two families develop regions where matter moves with velocities greater than the velocity of light. The second family includes the remarkable configuration of a black hole surrounded by a disk made of tachyonic matter up the edge, which is at the photonic orbit. In addition some configurations have regions where the energy density is negative in violation of the weak energy condition. This is the analogue of the strut that holds two particles apart in Weyl solutions, and which has a negative energy density. The last family admits configurations which do not contain tachyonic regions and so has greater physical relevance. The disks of this family have an inner edge and a well-defined behavior at infinity. In the limit of a negligible disk mass one obtains the solution for an accretion (test-particle) disk.

Received 12 April 1993

DOI: https://doi.org/10.1103/PhysRevD.49.5135

Issue
Vol. 49, Iss. 10 — 15 May 1994

Sidenote:
22 of Patricio Letelier's 180 papers on INSPIRE have "disk(s)" in the title; this paper here is his most famous among them

superposition of two exact axi-symmetric stationary solutions: rotating black hole and thin disk of (counter-)rotating particles without angular momentum

numerous follow-up papers and extensions

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Goal: drop all assumptions about symmetries to describe generically interacting black holes

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Step back for a moment and consider simpler theory

Stepping back: How do describe electrodynamics?

- theoretically straightforward: just keep track of all the point sources, the electrons and ions, and use the appropriate Green function

$$\nabla^2 A^\mu(x^\nu) = \sum_{i=1}^{N} q_i \int \mathrm{d}\tau_i \, \frac{\mathrm{d}\bar{x}_i^\mu(\tau_i)}{\mathrm{d}\tau_i} \, \delta\big(x^\nu - \bar{x}_i^\nu(\tau_i)\big)$$

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> Intend to do something similar for black holes: impose different bc's at the black hole horizon!

Back to black holes: Near horizon boundary conditions

Near horizon expansion:

$$g_{\mu\nu} = \begin{pmatrix} -\kappa^2\rho^2 + \mathcal{O}(\rho^3) & \mathcal{O}(\rho^2) & f_{ta}\,\rho^2 + \mathcal{O}(\rho^3) \\ g_{\rho t} = g_{t\rho} & 1 + \mathcal{O}(\rho) & f_{\rho a}\,\rho + \mathcal{O}(\rho^2) \\ g_{at} = g_{ta} & g_{a\rho} = g_{\rho a} & \Omega_{ab} + \mathcal{O}(\rho^2) \end{pmatrix}$$

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Meaning of various quantities:

- t: time-coordinate
- ρ: radial coordinate; $\rho \to 0$: black hole horizon; $\rho > 0$: outside region
- x^a: transversal coordinates on horizon ("angular coordinates")

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- ρ: radial coordinate; $\rho \to 0$: black hole horizon; $\rho > 0$: outside region
- x^a: transversal coordinates on horizon ("angular coordinates")
- $\kappa(t,\, x^a) \neq 0$: surface gravity of black hole

Back to black holes: Near horizon boundary conditions

Near horizon expansion:

$$g_{\mu\nu} = \begin{pmatrix} -\kappa^2\rho^2 + \mathcal{O}(\rho^3) & \mathcal{O}(\rho^2) & f_{ta}\,\rho^2 + \mathcal{O}(\rho^3) \\ g_{\rho t} = g_{t\rho} & 1 + \mathcal{O}(\rho) & f_{\rho a}\,\rho + \mathcal{O}(\rho^2) \\ g_{at} = g_{ta} & g_{a\rho} = g_{\rho a} & \Omega_{ab} + \mathcal{O}(\rho^2) \end{pmatrix}$$

Keeping only leading terms: Rindler-like approximation:

$$\mathrm{d}s^2 = g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu = -\kappa^2\rho^2\,\mathrm{d}t^2 + \mathrm{d}\rho^2 + \Omega_{ab}\,\mathrm{d}x^a\,\mathrm{d}x^b + \dots$$

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- $\Omega_{ab}(t, x^a)$: transversal metric ("sphere")

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- $f_{\mu\nu}(t,\, x^a)$: additional functions characterizing interacting black hole
- Still need to specifiy what is fixed and what is allowed to vary!

Back to black holes: Near horizon boundary conditions

Near horizon expansion:

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- Simple possibility (canonical ensemble): keep fixed κ: $\delta\kappa = 0$

Fineprint: $\delta\kappa$ obeys a condition displayed later

Back to black holes: Near horizon boundary conditions

Near horizon expansion:

$$g_{\mu\nu} = \begin{pmatrix} -\kappa^2\rho^2 + \mathcal{O}(\rho^3) & \mathcal{O}(\rho^2) & f_{ta}\,\rho^2 + \mathcal{O}(\rho^3) \\ g_{\rho t} = g_{t\rho} & 1 + \mathcal{O}(\rho) & f_{\rho a}\,\rho + \mathcal{O}(\rho^2) \\ g_{at} = g_{ta} & g_{a\rho} = g_{\rho a} & \Omega_{ab} + \mathcal{O}(\rho^2) \end{pmatrix}$$

- Simple possibility (canonical ensemble): keep fixed κ: $\delta\kappa = 0$
- Other functions allowed to vary; define

$$\mathcal{P} = \sqrt{\Omega} \qquad \mathcal{J}_a = \sqrt{\Omega}\,\big(\partial_t f_{\rho a} - 2 f_{ta}\big)$$

allowing $\delta\mathcal{P} \neq 0$ and $\delta\mathcal{J}_a \neq 0$

Back to black holes: Near horizon boundary conditions

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$$\mathcal{P} = \sqrt{\Omega} \qquad\qquad \mathcal{J}_a = \sqrt{\Omega}\,\big(\partial_t f_{\rho a} - 2 f_{ta}\big)$$

  allowing $\delta\mathcal{P} \neq 0$ and $\delta\mathcal{J}_a \neq 0$
- implies allowed variations of metric given by

$$\delta g_{\mu\nu} = \begin{pmatrix} \mathcal{O}(\rho^3) & \mathcal{O}(\rho^2) & \delta f_{ta}\,\rho^2 + \mathcal{O}(\rho^3) \\ g_{\rho t} = g_{t\rho} & \mathcal{O}(\rho) & \delta f_{\rho a}\,\rho + \mathcal{O}(\rho^2) \\ g_{at} = g_{ta} & g_{a\rho} = g_{\rho a} & \delta\Omega_{ab} + \mathcal{O}(\rho^2) \end{pmatrix}$$

Back to black holes: Near horizon boundary conditions

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- near horizon expansion preserved by ∞ near horizon Killing vectors

$$\mathcal{L}_\xi g_{\mu\nu} = \mathcal{O}(\delta g_{\mu\nu}) \qquad\qquad \xi = \eta(t,\,x^b)/\kappa\,\partial_t + \eta^a(t,\,x^b)\,\partial_a + \mathcal{O}(\rho)$$

Near horizon symmetries

Remarkable result: infinitely many near horizon symmetries

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Associated Noether-like co-dimension-2 charges:

$$\delta Q[\eta,\,\eta^a] = \int \mathrm{d}x^{D-2}\,\big(\eta\,\delta\mathcal{P} + \eta^a\,\delta\mathcal{J}_a\big)$$

Variations determined from near horizon Killing equations

$$\delta_{\eta,\eta^a}\mathcal{P} = \eta^a\partial_a\mathcal{P} + \mathcal{P}\,\partial_a\eta^a \qquad \text{scalar density}$$

$$\delta_{\eta,\eta^a}\mathcal{J}_a = \mathcal{P}\,\partial_a\eta + \eta^c\partial_c\mathcal{J}_a + \mathcal{J}_c\,\partial_a\eta^c + \mathcal{J}_a\,\partial_c\eta^c \qquad \text{1-form density}$$

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If you see such a result for the first time:

- analogous to Gauss law and electric charge in electrodynamics, but infinitely many charges
- derivable using canonical or covariant methods
  see e.g. lecture notes Compère, Fiorucci '17
- main difference to global Noether charges: co-dimension-2 rather than co-dimension-1 (surface integral, not volume integral)
- only get variation of charges, not yet the charges themselves
- charges generate Poisson bracket algebra:

$$\delta_{\eta_1^i} Q[\eta_2^i] = \{Q[\eta_1^i],\,Q[\eta_2^i]\} = Q[\eta_1^i \circ \eta_2^i] + Z_{12}$$

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Still need to make choices!

- (how) do chemical potentials η, η^a depend on charges $\mathcal{P}, \mathcal{J}_a$?

Remarkable result: infinitely many near horizon symmetries

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- straightforward choice: η, η^a independend from charges
  Donnay, Giribet, González, Pino '15

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Discuss two examples on next slides

Example 1: BMS near horizon symmetries

DG, Pérez, Sheikh-Jabbari, Troncoso, Zwikel '19

- Choose

$$\eta = \eta_{(s)} \mathcal{P}^{s/(D-2)} \qquad \delta\eta_{(s)} = 0 = \delta\eta^a$$

Fineprint: a necessary condition is

$$\delta\kappa = \partial_t \eta + \eta^a \partial_a \kappa$$

so $\delta\kappa = 0$ may not be achievable

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DG, Pérez, Sheikh-Jabbari, Troncoso, Zwikel '19

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$$\eta = \eta_{(s)} \mathcal{P}^{s/(D-2)} \qquad \delta\eta_{(s)} = 0 = \delta\eta^a$$

- Obtain charges ($\mathcal{P}_{(s)} \sim \mathcal{P}^{s/(D-2)+1}$)

$$Q[\eta_{(s)},\, \eta^a] = \int \mathrm{d}^{D-2}x \,\big(\eta_{(s)} \mathcal{P}_{(s)} + \eta^a \mathcal{J}_a\big)$$

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- Determine Poisson bracket algebra from $\{Q[\eta_1^i],\, Q[\eta_2^i]\} = \delta_{\eta_1^i} Q[\eta_2^i]$

$$\{\mathcal{J}_a(x),\, \mathcal{P}_{(s)}(y)\} = \Big(\frac{s}{D-2}\,\mathcal{P}_{(s)}(y)\,\frac{\partial}{\partial x^a} - \mathcal{P}_{(s)}(x)\,\frac{\partial}{\partial y^a}\Big)\,\delta^{(D-2)}(x-y)$$

$$\{\mathcal{P}_{(s)}(x),\, \mathcal{P}_{(s)}(y)\} = 0$$ **supertranslations**

$$\{\mathcal{J}_a(x),\, \mathcal{J}_b(y)\} = \Big(\mathcal{J}_a(y)\,\frac{\partial}{\partial x^b} - \mathcal{J}_b(x)\,\frac{\partial}{\partial y^a}\Big)\,\delta^{(D-2)}(x-y)$$

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- For $s = 0$: recover results by Donnay et al. '15

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supertranslations

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- For $s=0$: recover results by Donnay et al. '15
- For $s=1$ and $D=4$: famous BMS-algebra Bondi et al.; Sachs '62 but here this algebra appears near the horizon, not at infinity!

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- For $s=0$: recover results by Donnay et al. '15
- For $s=1$ and $D=4$: famous BMS-algebra Bondi et al.; Sachs '62
- Otherwise: spin-s generalization of BMS in arbitrary D

Example 2: Soft Heisenberg hair

DG, Pérez, Sheikh-Jabbari, Troncoso, Zwikel '19

- Choose

$$\eta^a = \eta^a_{\mathrm{H}}\,\mathcal{P}^{-1} \qquad \eta = \eta_{\mathrm{H}} - \eta^a_{\mathrm{H}}\,\mathcal{J}_a\,\mathcal{P}^{-2} \qquad \delta\eta^a_{\mathrm{H}} = \delta\eta_{\mathrm{H}} = 0$$

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- Obtain charges ($\mathcal{J}^{\rm H}_a = \mathcal{J}_a\mathcal{P}^{-1}$)

$$Q[\eta_{\rm H},\,\eta^a_{\rm H}] = \int \mathrm{d}^{D-2}x\,\big(\eta_{\rm H}\mathcal{P} + \eta^a_{\rm H}\mathcal{J}^{\rm H}_a\big)$$

Note: $\mathcal{J}^{\rm H}_a$ is now a 1-form rather than a 1-form density

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- Obtain charges ($\mathcal{J}^{\mathrm{H}}_a = \mathcal{J}_a\mathcal{P}^{-1}$)

$$Q[\eta_{\mathrm{H}},\,\eta^a_{\mathrm{H}}] = \int \mathrm{d}^{D-2}x\,\big(\eta_{\mathrm{H}}\mathcal{P} + \eta^a_{\mathrm{H}}\mathcal{J}^{\mathrm{H}}_a\big)$$

- Determine Poisson bracket algebra from $\{Q[\eta^i_1],\,Q[\eta^i_2]\} = \delta_{\eta^i_1}Q[\eta^i_2]$

$$\{\mathcal{J}^{\mathrm{H}}_a(x),\,\mathcal{P}(y)\} = \frac{\partial}{\partial x^a}\,\delta^{(D-2)}(x-y)$$ **central term**

$$\{\mathcal{P}(x),\,\mathcal{P}(y)\} = 0$$ **supertranslations**

$$\{\mathcal{J}^{\mathrm{H}}_a(x),\,\mathcal{J}^{\mathrm{H}}_b(y)\} = \mathcal{P}^{-1}(x)\,F_{ba}(x)\,\delta^{(D-2)}(x-y)$$

with $F_{ab} := \partial_a\mathcal{J}^{\mathrm{H}}_b - \partial_b\mathcal{J}^{\mathrm{H}}_a$

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- Obtain charges ($\mathcal{J}^{\rm H}_a = \mathcal{J}_a\mathcal{P}^{-1}$)

$$Q[\eta_{\rm H},\,\eta^a_{\rm H}] = \int \mathrm{d}^{D-2}x\,\big(\eta_{\rm H}\mathcal{P} + \eta^a_{\rm H}\mathcal{J}^{\rm H}_a\big)$$

- Determine Poisson bracket algebra from $\{Q[\eta^i_1],\,Q[\eta^i_2]\} = \delta_{\eta^i_1}Q[\eta^i_2]$

$$\{\mathcal{J}^{\rm H}_a(x),\,\mathcal{P}(y)\} = \frac{\partial}{\partial x^a}\,\delta^{(D-2)}(x-y) \qquad \textbf{central term}$$

$$\{\mathcal{P}(x),\,\mathcal{P}(y)\} = 0 \qquad \textbf{supertranslations}$$

$$\{\mathcal{J}^{\rm H}_a(x),\,\mathcal{J}^{\rm H}_b(y)\} = 0$$

- for $D=3$ and non-rotating black holes (Schwarzschild): $F_{ab} = 0$

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$$Q[\eta_{\rm H},\,\eta^a_{\rm H}] = \int \mathrm{d}^{D-2}x\,\big(\eta_{\rm H}\mathcal{P} + \eta^a_{\rm H}\mathcal{J}^{\rm H}_a\big)$$

- Determine Poisson bracket algebra from $\{Q[\eta^i_1],\,Q[\eta^i_2]\} = \delta_{\eta^i_1}Q[\eta^i_2]$

$$\{\mathcal{Q}(x),\,\mathcal{P}(y)\} = \delta^{D-2}(x-y) \qquad \textbf{Heisenberg algebra}$$
$$\{\mathcal{P}(x),\,\mathcal{P}(y)\} = 0$$
$$\{\mathcal{Q}(x),\,\mathcal{Q}(y)\} = 0$$

- for $D = 3$ and non-rotating black holes (Schwarzschild): $F_{ab} = 0$
- locally: $\mathcal{J}^{\rm H}_a = \partial_a\mathcal{Q}$

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DG, Pérez, Sheikh-Jabbari, Troncoso, Zwikel '19

- Choose

$$\eta^a = \eta^a_{\mathrm{H}}\,\mathcal{P}^{-1} \qquad \eta = \eta_{\mathrm{H}} - \eta^a_{\mathrm{H}}\,\mathcal{J}_a\,\mathcal{P}^{-2} \qquad \delta\eta^a_{\mathrm{H}} = \delta\eta_{\mathrm{H}} = 0$$

- Obtain charges ($\mathcal{J}^{\mathrm{H}}_a = \mathcal{J}_a\mathcal{P}^{-1}$)

$$Q[\eta_{\mathrm{H}},\,\eta^a_{\mathrm{H}}] = \int \mathrm{d}^{D-2}x\,\big(\eta_{\mathrm{H}}\mathcal{P} + \eta^a_{\mathrm{H}}\mathcal{J}^{\mathrm{H}}_a\big)$$

- Determine Poisson bracket algebra from $\{Q[\eta^i_1],\,Q[\eta^i_2]\} = \delta_{\eta^i_1}Q[\eta^i_2]$

$$\{\mathcal{Q}(x),\,\mathcal{P}(y)\} = \delta^{D-2}(x-y)$$ **Heisenberg algebra**

$$\{\mathcal{P}(x),\,\mathcal{P}(y)\} = 0$$

$$\{\mathcal{Q}(x),\,\mathcal{Q}(y)\} = 0$$

- for $D=3$ and non-rotating black holes (Schwarzschild): $F_{ab} = 0$
- locally: $\mathcal{J}^{\mathrm{H}}_a = \partial_a\mathcal{Q}$
- suprisingly simple algebra!

Example 2: Soft Heisenberg hair

DG, Pérez, Sheikh-Jabbari, Troncoso, Zwikel '19

- Why **"soft Heisenberg hair"**?

  Historical notes:
  - expression "soft hair" coined by Hawking in his last series of papers, together with Perry and Strominger in 2015-16
  - soft hair as part of near horizon symmetries found by Donnay, Giribet, González, Pino '15
  - soft Heisenberg hair found first in $D = 3$ by Afshar, Detournay, DG, Merbis, Peréz, Tempo, Troncoso '16
  - generalized to higher dimensions, higher spins, higher derivative theories, dynamical black holes, ... '16-'21

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- near horizon Hamiltonian

$$H = Q[\partial_t] = \kappa\,\mathcal{P}_0$$

commutes with all generators of near horizon symmetry algebra!

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- energy excited states unchanged!

$$H\mathcal{Q}|\psi\rangle = \mathcal{Q}H|\psi\rangle = E\mathcal{Q}|\psi\rangle \qquad\qquad H\mathcal{P}|\psi\rangle = \mathcal{P}H|\psi\rangle = E\mathcal{P}|\psi\rangle$$

hence, excitations are soft!

Near horizon entropy law

- Bekenstein–Hawking entropy suprisingly simple in terms of near horizon charges:

$$S = \frac{A}{4} = 2\pi \, \mathcal{P}_0$$

with $\mathcal{P}_0 = \int \mathrm{d}^{D-2}x \, \mathcal{P}(x)$

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- in case you know the Cardy-formula: this is its near horizon version!

technical details for the experts: in $D=3$ near horizon charges related to asymptotic Virasoro charges through (twisted) Sugawara construction

$$\mathcal{L}\sim\frac{6}{c}\,\mathcal{P}^2+i\mathcal{P}'$$

Cardy-formula:

$$S\sim 2\pi\,\sqrt{\frac{c\mathcal{L}_0}{6}}=2\pi\,\mathcal{P}_0$$

similar constructions work for higher spin black holes

Summary & Outlook

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$\text{BMS}_D^{(s)}$ algebra for higher spins s appeared recently in context of higher spin theories Campoleoni, Francia, Heissenberg '17-'21

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  Adami, DG, Sheikh-Jabbari, Taghiloo, Yavartanoo, Zwikel '21

$$\frac{\mathrm{d}Q}{\mathrm{d}t} \sim -\text{flux}$$

  like in asymptotic region with "leaky boundary conditions"

  (Compére), Fiorucci, Ruzziconi '19-'21

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- speculation: near horizon soft hair encodes black hole microstates construction in 3d Afshar, DG, Sheikh-Jabbari, (Yavartanoo) '16 ('17)

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A lot remains to be discovered (perhaps by you)!

Last words

We have come a long way since pioneering work by Letelier et al to describe interacting black holes

Our perspective: put physics into choice of near horizon boundary conditions, like in macroscopic electrodynamics

Last words

https://indico.cern.ch/event/1085701/

Thanks for your attention!

MUITO OBRIGADO!