Induced action in QCD[‡]

Victor Novozhilov and Yuri Novozhilov
V. A. Fock Department of Theoretical Physics
St. Petersburg State University 198904, St. Petersburg, Russia
e-mail: `yunovo@pobox.spbu.ru, novozhil@nvv.gc.spb.ru`

Abstract. We consider the problem of fermionic composites in the strong coupling regime of the low mass scale and not-so-low mass scale QCD. The group-theoretical approach permits easily to identify those collective variables which describe composite particles in the lowest scale part of QCD. When scale increases and in absence of new approximate symmetry, we propose to study possibilities opened by induced actions for background fields with finite composite scale which acquire dynamics due to vacuum fluctuations of quarks. We investigated an induced action for scalar mesons and found that their masses can be enhanced above double quark mass, due to condensates.

Keywords: quantum anomalies, effective lagrangians

Mathematics Subject Classifications (2000): 81T50,81T10

Dedicated to the memory of Moshé Flato

We first met Moshé in Paris. For one of us it happened during his short stay at Orsay in 1969, the other was brought to Moshé in 1974 by his father, who at that time was "le Directeur à l'UNESCO."

Moshé was a phenomenon. Nature gave him so many personal qualities at extreme that rarely can be found in one human being. His energy was legendary. He was exceptionally quick-minded and intelligent. Moshé irradiated such warmth and compassion that the moment you met him you become his friend.

People loved Moshé not only because he was sympathetic, clever and really truthful to friends, but also because he belonged to the type of personalities who are so needed for active life of the scientific community. His attitude to Science was close a religious one. He was always ready to fight for Science, he considered the scientific community as a great family and felt himself responsible for its health. People admired his way of doing it with such dedication and frankness, which, they felt, they were not able to exercise themselves.

LMP was one of his creations, together with Daniel. They were first to realize that such type of journal was needed. One can find in LMP the beginning of many research lines, and among them, one of the first presentation of ideas summarized below.

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1. Introduction

The group-theoretical approach to composite particles in QCD starts from quantum anomalies. Adler-Bardeen-Gross-Jackiw chiral anomalies [1, 2, 3] were first studied perturbatively in terms of Feynman diagrams. In the formalism of functional integrals these anomalies can be obtained by integration over fermion fields [4]. Because of anomalies, the invariance properties of the quantum theory differ from those of the same theory at the classical level. As it was shown by Fujikawa [4], an anomaly arises when the classical action is invariant under the chiral group $\mathscr{H}$ of transformations, while the functional measure is not. The anomaly can be integrated up to a chiral action, which can be divided into topological and normal-parity parts. The topological part (the Wess-Zumino-Witten term [5, 6]) presents an obstruction to renormalization. However, this part can be eliminated by choosing a special set of fermionic representations. The normal-parity part represented by polynomials of fields gives the first approximation to the low energy effective Lagrangian. Thus, the chiral action summarizes all non-invariance effects of the functional integral with respect to $\mathscr{H}$.

The conformal anomaly arises as a response of bosonic and fermionic functional integrals to conformal transformations of fields in the classical action associated with the transformation $g\colon\ g_{\mu\nu} \to g_{\mu\nu}\exp(2\sigma(x))$ [7] of an external metric $g_{\mu\nu}$. By integrating the conformal anomaly in the case of fermionic and/or scalar fields, one can obtain the conformal action (while the case of gauge fields requires a special study of gauge independence [8]), which summarizes all non-invariance effects of the functional integral with respect to the conformal group.

In the group-theoretical approach both cases can be treated in a similar manner, because what is only important is a possibility to subdivide the functional integral into two factors, $Z = Z_{inv}Z_{non}$, where Z_{inv} is invariant, while Z_{non} is non-invariant under the transformations of a group $\mathscr{G}$ with the invariant Haar measure. Then invariant factors Z_{inv} and Z_{inv}^{-1} can be built separately by transforming $Z \to Z^g$, $g \in \mathscr{G}$, and integrating over all group transformations $g(x)$ — $Z_{inv} = \int DgZ^g$, or $Z_{inv}^{-1} = \int Dg\,(Z^g)^{-1}$ [9, 10]. An effective action $W(g)$ for a field $g(x)$ is defined by

$$Z_{non} \equiv \int Dg \exp iW(g)\,, \quad W(g) = -i\ln\left(\frac{Z}{Z^g}\right) \tag{1}$$

up to $\mathscr{G}$-invariant terms. Note that this definition does not require $\mathscr{G}$-invariance of the classical Lagrangian. Z_{non} can be considered as a functional integral for a quantum field $g(x)$ with the classical action $W(g)$ which describes symmetry breaking, i.e., chiral symmetry breaking or breaking of the conformal symmetry. Thus, it is reasonable to use such a description of composites in the scale region where the chiral and conformal symmetries breaking dominate in the physical picture (the lowest mass scale region). Characteristic phenomenological scales are given by the quark C_q and gluon C_g condensates, $C_q = -(180\text{ MeV})^3$, $C_g = (330\text{ MeV})^4$.

The group-theoretical approach was used for a systematic derivation of effective Lagrangians in the following cases:

(a) For the chiral field, $g \to U = \exp(i\Pi)$, where the pseudoscalar field Π describes pions and kaons [10];
(b) For an extended chiral field $g \to \mathscr{U}$ of the extended chiral group $E\chi$ describing pseudoscalar mesons and diquarks [11];
(c) For the dilaton field $\sigma(x)$, where $g \to \exp(\sigma)$ is related to the conformal transformation $g_{\mu\nu} \to (\exp 2\sigma) g_{\mu\nu}$ [12].

In this way we get quantum composite fields $U, \mathscr{U}$, and σ with their vacuum functionals, i.e., with integration over these fields.

The induced action for a background field describes its dynamics arising due to interaction with vacuum fluctuations of quantum fields. The question is to what extent the induced action approach can be used as a kind of extension of the group-theoretical approach into not-so-low mass scale region of QCD, when we include impact of quark and gluon condensates into an induced action and correlate the compositeness scale with the chiral breaking scale. One of the problem to solve is how do mesons in not-so-low scale region get masses exceeding two constituent quark masses (excited mesons).

In this paper we consider the simplest model of quantized quark fields in a background of scalar field and investigate properties of dynamical scalar (classical) fields resulting after integration over quarks and application of the compositeness scheme.

2. Quantum field in classical background and classical action for composite field

2.1. Induced classical gravity

The first induced action for background field has been considered in relation with gravity. Quantum matter fields on curved space represent the most general example of such class. Vacuum fluctuations of matter fields transform a given metric into a classical dynamical gravitational field with the Einstein action in the long-wave region which got the name of 'Induced Gravity'. This fact was discovered by Zeldovich and Sakharov [13] and developments were reviewed by Adler [14]. Due to an ambiguity [15] in the definition of the Newton constant, Induced Gravity was forgotten for a decade and then reconsidered [16] with modern computational tools in the framework of bosonization approach. "Einsteinization" [17] of an external metric under the influence of collective vacuum mode of matter fields represents a phenomenon of 'partial compositeness', when quantum fields do not create a new field, but only provide an existing field with a kinetic term and interactions.

Let $L\left(\Psi, g_{\mu\nu}\right)$ be a Lagrangian of matter fields Ψ on curved background $g_{\mu\nu}$ and let $Z_\Psi\left[g_{\mu\nu}\right]$ denote the vacuum functional by

$$Z_{\Psi}\left[g_{\mu\nu}\right] = \int D\Psi \exp\left(\int dx\, L\left(\Psi, g_{\mu\nu}\right)\right). \quad (2)$$

Integration over matter Ψ in Z_{Ψ} takes into account vacuum fluctuations of the fields Ψ, which provide an external metric $g_{\mu\nu}$ by kinetic term and interactions described by an effective action $W\left(g_{\mu\nu}\right)$

$$\exp\left(iW\left(g_{\mu\nu}\right)\right) \equiv Z_{\Psi}\left[g_{\mu\nu}\right]. \quad (3)$$

It is the action $W\left(g_{\mu\nu}\right)$ that describes induced Einsteinian gravity if the evaluation of Z_{Ψ} is implemented carefully.

2.2. Scalar mesons

As an example, consider a quantum quark field ψ and in background a scalar field $S(x)$ which serves as a collective variable in a model of scalar mesons [18]. A model of flavor vector mesons was constructed earlier in a similar approach [19].

The purpose of the model is to develop a theory of isotriplet and isosinglet scalar mesons a_0 and f_0. Description of the low energy QCD in terms of the quark and gluon condensates only presupposes that the coupling constant α_S is frozen up to the chiral breaking scale. Such an approximation is not valid for higher scales. We propose to describe QCD physics for higher scales, and, in particular, scalar mesons by the composite model with the finite compositeness scale. We use an approach proposed for the model of composite electroweak bosons [20] and a definition of a composite particle in renormalization theory through radiative corrections [21].We use also similarity between low energy scalars and Higgs fields when gauge fields are also present.

Integration over fermions in the quark path integral in its simplest form is equivalent to one-loop approximation which produces polynomials of scalar fields and, consequently, may lead to scalar condensate. An advantage of the path integral approach is that it can easily take into account the quark and gluon condensates [10] and incorporate the finite compositeness scale.

Our aim is to obtain an effective Lagrangian for a dynamical composite field $\widehat{S}(x)$ through radiation corrections to the Dirac Lagrangian with an external field $S(x)$. A composite field $\widehat{S}(x)$ is defined by vanishing of its renormalization constant $z_S\left(\frac{\Lambda}{\mu}\right) \to 0$ when μ reaches the compositeness scale Λ [21]. We consider composites in the approximation of fermion loops resulting from integration over fermions in the path integrals. If $S(x)$ were elementary, its renormalization constant would be $z_S = 1 + \Delta$, where Δ contains logarithms.For a composite $\widehat{S}(x)$ suitable normalization leads to $z_S = \Delta$.

The Lagrangian for fermions interacting with a scalar background S and the gluon field $G_\nu = (g/2i)\, G^k_\nu \lambda_k$, and the corresponding path integral are given by

$$\begin{gathered} L = \overline{\psi}\,(\not{D} + fS)\,\psi, \\ \mathbf{Z}[S] = \exp iW\,(S) = \int D\overline{\psi} D\psi \exp\{i \int dx\, L\} \equiv Z_\psi\,[D+S]\,, \end{gathered} \tag{4}$$

where D is the Dirac operator for massless fermions with covariant derivative D_μ containing the gluon field and f is the Yukawa coupling constant. The fermion path integral $\mathbf{Z}[S]$ defines an induced action $W(S)$ for S in presence of the gluon field. In general, the scalar background $S(x)$ should be introduced into the Lagrangian together with a weight function $\Omega(S)$ independent of fermions, $L \to L + \Omega$, which arises from other interactions of scalar S. It is understood that all expressions for $\mathbf{Z}$ are averaged over gluons G_μ.

The quark spectrum in the gluon vacuum displays non-trivial features [22] which have escaped until now from detailed analysis and are known by their manifestation in condensation process. The quark condensate

$$C_q = \langle \overline{\psi}(x)\, \psi(x) \rangle_{\overline{\psi},\psi,G} = -i \left\langle \frac{\delta \ln \mathbf{Z}}{\delta f S}\, |_{S=0} \right\rangle_G \tag{5}$$

is the vacuum expectation value obtained by averaging over both quark and gluon fields. Its numerical value is known from experiments. The origin of the quark condensate may be due to a gap in the quark spectrum density at the origin [23] or to an asymmetry of the spectral density [10, 24]. We shall deal with the quark spectrum in the Θ-function in a phenomenological way: We do not take into account the spectral gap or the asymmetry, but just integrate this expression for a given C_q and add the condensate term $C_q tr_f S$ to the induced Lagrangian $L(S,\Lambda)$ in the Minkowski space

$$\begin{aligned} L(S,\Lambda) - L(S,\mu) = & a\sigma f^2 tr_f \left(D_\mu S\right)^2 + 6a\sigma\xi_2 f^2 \Lambda^2 tr_f S^2 \\ & - a\sigma f^4 tr_f S^4 - 3\sigma\xi_3 f C_q tr_f S - \sigma \frac{N_f}{24\pi^2} tr_C G^2_{\mu\nu}, \end{aligned} \tag{6}$$

where $G_{\mu\nu}$ is the gluonic field strength, ξ_n describes running of dimensionful coefficients

$$\xi_n = \frac{1 - e^{-n\sigma}}{n\sigma}, \quad \sigma = \ln \frac{\Lambda}{\mu}, \tag{7}$$

tr_f and tr_C refer to flavor and color matrices. $a = N_C/8\pi^2$, N_C is the number of colors. In (6) the kinetic energy is positive for $\sigma > 0$ and disappears at $\sigma = 0$. It follows from (6) that the induced potentials $U(S,\Lambda)$ and $U(S,\mu)$ include S^2- and S^4-terms; the S^2-term enters with a wrong sign, so that the potential may have a minimum at S_0 corresponding to formation of scalar condensate. We choose S_0 as the lower scale of the region under consideration with the compositeness scale

Λ_S as the upper scale. The normalization scale μ is arbitrary within the region $S_0 \leq \mu \leq \Lambda_S$.

Note that with the specification of the path integral by an invariant momentum cut-off, the Lagrangian (6) does not contain terms with negative powers of the cut-off Λ. The term proportional to $\Lambda^2 - \mu^2$ is absent, because it is always understood that the free fermion part is eliminated.

With the standard kinetic term in the form $\frac{1}{4} tr_f \left(D_\mu S_r \right)^2$ for the renormalized field S_r we get the following renormalization constant and couplings

$$z_S = 4a\sigma f^2, \quad m^2 = \frac{3}{2}\xi_2 \Lambda^2, \quad \lambda = \frac{1}{4a\sigma}, \quad \eta = 3\sqrt{\frac{\sigma}{a}}\xi_3 \tag{8}$$

for the potential

$$U^r_{eff} = -m^2 tr S_r^2 + \frac{\lambda}{4} tr S_r^4 - \eta C_q tr S_r. \tag{9}$$

Thus, at the compositeness scale $\sigma \to 0$, the coupling λ is diverging, m^2 remains finite, while η becomes inessential. By neglecting η we would get for a flavor singlet S_r the minimum of U^r_{eff} for $\sigma \to 0$ at $S_r^2 = (3N_C/4\pi^2)\Lambda^2\sigma \ll \Lambda^2$, i.e., well below the upper scale.

In order to study composite scalars far from the compositeness scale, one should use the induced action $W(S,\mu)$. The easiest way to find $W(S,\mu)$ is to calculate separately the potential and the kinetic terms. The potential (even powers of S and logarithms) can be calculated exactly; the quark condensate term will be neglected. For the kinetic term one can use an expansion in the derivatives. We get for the induced Lagrangian

$$\begin{aligned} L(S,\mu) = & \frac{1}{2} a f^2 tr_f \ln(\frac{\mu^2}{f^2 S^2}) (D_\nu S)^2 + 3 a f^2 tr_f \left(\mu^2 - f^2 S^2\right) S^2 - \frac{1}{4} a f^4 tr_f S^4 \\ & - \frac{1}{2} a f^4 tr_f S^4 \ln(\frac{\mu^2}{f^2 S^2}) - \frac{N_f}{48\pi^2} tr_C (G^2_{\mu\nu}) tr_f \ln\left(\frac{\mu^2}{f^2 S^2}\right), \end{aligned} \tag{10}$$

under the condition $\Lambda_S^2 \geq \mu^2 \geq f^2 S^2$. For the compositeness scale $\mu = \Lambda_S$, the induced potential $U\left(S, \Lambda_S\right)$ will have a minimum at a point $S = S_0$ defining the lowest scale in the renormalization constant $z_S = 2af^2 \ln\left(\Lambda_S^2 / f^2 S_0^2\right)$ for $L(S, \Lambda_S)$.

In the model of induced action with the finite compositeness scale Λ_s, we assune that $L(S,\mu) = L(\mu, \Lambda_s)$ and consider

$$L(S,\Lambda_s) = \frac{1}{2}(L(S,\mu) + L(\mu,\Lambda_s)) \tag{11}$$

as an effective action for the model.

3. Nonlinear representation of the composite scalar field

The most interesting case is when the composite scalar field develops condensate and interacts with the quark condensate. Therefore, we consider a nonlinear representation for $S(x)$ and denote it, in this case, by $\Phi = \Phi_0 \exp\phi$, where Φ_0 is a large slowly varying field and $\phi = \phi_0 + \phi_k \tau_k$ are small fluctuations; τ_k are the Pauli matrices for isospin.

The induced Lagrangian for the renormalized composite field $\Phi_r = z_\phi^{1/2}\Phi$, $z_\phi = z_S$, and the compositeness scale Λ_s as the higher scale μ is given by

$$L(\Phi_r, \Lambda_S) = \frac{1}{4} tr_f (D_\nu \Phi_r)^2 - U\left(\Phi_r, \Lambda_S\right), \quad a = 0, k, \tag{12}$$

where U is the induced potential with positive coefficients $m_0^2, \lambda, \zeta, \varepsilon^4$ defined at $\mu = \Lambda$:

$$U\left(\Phi_r, \Lambda_S\right) = -m_0^2 tr_f \Phi_r^2 + \frac{\lambda_0}{4} tr_f \Phi_r^4 + \frac{\zeta}{4} tr_f \Phi_r^4 \ln\left(\frac{\Lambda_S^2}{f^2\Phi_r^2}\right) + \varepsilon^4 \ln\frac{\Lambda_S^2}{f^2\Phi_r^2}, \tag{13}$$

with the minimum at Φ_0 defined by the equations

$$\left(2\lambda_0 - \zeta\right)\Phi_0^4 + 2\zeta\Phi_0^4 \ln\frac{\Lambda_S^2}{\Phi_0^2} - 4m_0^2\Phi_0^2 - 4\varepsilon^4 = 0, \tag{14a}$$

$$(2\lambda_0 - 3\zeta + 2\zeta\ln\frac{\Lambda_S^2}{\Phi_0^2})\Phi_0^4 + \varepsilon^4 \succ 0, \tag{14b}$$

and the mass M of the composite field by $M^2 = 8\left(\lambda\Phi_0^2 - m_0^2\right)$.

In this relation the mass M does not depend explicitly on the gluon condensate coefficient ε^4 and on the coefficient ζ. An expression for the mass looks as if these two coefficients are zero. The relation between the mass of composite field and the quark mass $z_\phi M^2/\Phi_0^2$ is a function of the parameter $\chi^2 = \Lambda_S^2/\Phi_0^2$ that behaves like $(\ln\chi)^{-1}$. Thus, it is possible to have χ larger than 2. It means that two quarks can form a composite scalar field with a mass higher than the double quark mass due to contribution from gluons through the condensate. It can be seen explicitly when we can disregard terms with the coefficient ζ, i.e., when approximately $\Lambda_S^4 = e\Phi_0^4$. Then $\lambda\Phi_0^2 - m_0^2 = \left(m_0^4 + 2\lambda\varepsilon^4\right)^{1/2}$.

4. Conclusions

In the induced action approach, classical external fields in the quantum action get kinetic terms and interactions after integration over quantum fields. Such dynamical fields can be considered as composite fields formed by quantized fields integrated out. Induced actions obtained in this way may contain usual ultraviolet and infrared singularities.

We have considered a model of the composite scalar field induced by integration over quarks on background of classical scalar field with the same quantum numbers. Induced action admits formation of scalar condensate that plays a role at the lowest scale, thus preventing appearance of infrared singularities. In order to model the behaviour of the QCD coupling constant in the region with scalar mesons, we propose to introduce a finite compositeness scale as a phenomenological parameter. Contribution from the gluon condensate enhances the mass of the composite scalar field above two quark masses.

In realistic cases one should consider weight functions for external background scalars arising from other interactions as well as composite axial vector mesons together with contribution from the pure QCD sector.

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