# Are there macroscopic quantum bodies?

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**Abstract.** The validity of quantum laws in the macroscopic scale is discussed. Some size and time distortions predicted by quantum control theory can imitate the effects of the $\hbar$-deformation, helping us to observe the quantum behavior of huge micro-objects.

**Keywords:** Schrödinger's paradox, quantum control, time distortion

**Mathematics Subject Classifications (2000):** 81P99, 81Q05

> La physique est en effet une science qui n'avance que lorsqu'elle rencontre un paradoxe...
>
> Moshé Flato
> *Le pouvoir des mathématiques*
> *(Hachette 1990, p. 44)*

## 1. Introduction

One of puzzles of the present day physical theory is the fundamental difference between the classical and quantum structures. While the classical observables are just c-number functions on the classical phase space, the quantum ones form non-commutative operator algebras [1, 2, 3, 4]. The difference is especially critical on the conceptual level, in problems concerning measurement theory. It is enough to remember that one of the crucial points in the Copenhagen interpretation was the fundamental distinction between the micro-objects and the macroscopic measuring devices [5, 6, 2, 7], an idea which has raised multiple discussions. Indeed, should the measuring devices obey certain exceptional laws just because they were created to measure something? What are they if not massive bodies (agglomerations of micro-objects) sensitive to the presence of quanta? Should therefore the laws of quantum mechanics be not applicable to certain macroscopic bodies?

The orthodox interpretations (including the Copenhagen school [5, 2, 6]) try to disarm the paradox by arguments similar to the "double thinking" of George Orwell [8]. On one hand, they insist that $(A)$ the quantum theory applies to arbitrarily huge physical systems (including the entire universe). On the other hand, they tell you, that $(B)$ you should not apply it to the measuring devices, which are essentially classical. Thus, when trying to understand the measurement process, we should forget completely that the measuring device is a huge quantum system (or at


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least, we must keep this thought in the back of our minds). The motive persistently returns: When remembering $(A)$ you must forget $(B)$, and when remembering $(B)$ you must forget $(A)$.

The interpretational strategy would be almost successful were it not for the opposite trend which tries to exhibit the depth of the problem. Indeed, by treating seriously the linear evolution laws for quite arbitrary systems, one unavoidably ends up with macroscopic bodies which should be in the superposed states corresponding to different macroscopic effects. Thus, there should exist an undecided macroscopic observer in a superposed state of 'perceiving two different measurement results' on his macroscopic apparatus (and moreover, contemplating them both in his superposed brain). The most dramatic example was given by Erwin Schrödinger [9] in 1935, by considering an animal (a cat) in a superposed state of being alive and being dead (Schrödinger, like Einstein, opposed the philosophy of the Copenhagen school [7, 10, 11]). Yet, should such states be forbidden? Are they the physical reality or an exaggeration of the doctrine? This last possibility is considered by Ghirardi, Rimini, Grassi, Diossi [15, 16, 17], and Roger Penrose [18, 19], who believe that the quantum linearity should break for heavy objects.

The problem, in a sense, can serve to classify the schools of thinking. Some of them are aware of the creative role of the paradox, and henceforth, enjoy the difficulty, even if provoking *un vent de panique dans la corporation* (M. Flato [20], p. 44). The others try to evade the question (arguing that the physicist should think about graphs, not about metaphysical puzzles!)

Thanks to new empirical techniques, however, the puzzle is again in the center of discussion.

In a sequence of papers on the "Schrödinger's cat states" the authors try to construct the mesoscopic states sharing the situation of the Schrödinger cat. Until now, no size or mass limits are noticed; the unique challenge appears to be the decoherence phenomenon [21, 22]. An independent search in the neutron interferometry has proved the reality of the quantum superposition for the neutrons with the wave packet components separated by distances of the order of magnitude of centimeters [24]. As interesting are the effects of collective tunneling on the Josephson junction (see, e.g. Caldeira and Legget [25, 26]). A remarkable event of the nineties was the experimental observation of the interference pictures for sodium and beryllium atoms [27]. In the subsequent interference experiments with beams of $Na_2$ molecules, the authors could report: "here we coherently manipulate exclusively the center-of-mass motion" [28]. More striking even were the analogous diffraction experiments for large (about 6nm), weakly bound, fragile $He_2$ molecules [29] which, unless they break when passing through the gratings, show the interference patterns corresponding to the de Broglie waves of their mass centers. While the interference of atoms is already explored to construct gradiometers [30], all previous results were left behind by the observations of interference patterns for the soccer ball-shaped molecules $C_{60}$, passing with velocities about 200m/sec through a system of slits 50nm wide, 100nm apart [31].

In all these experiments the Planck constant is fixed. Yet, the effects are curiously related to the subject of deformation [32, 33, 34, 35, 36, 37]. Its main point is a "comparative anatomy": Our universe is best understood if submerged in an ample class of *virtual universes* where the physical laws could be altered and physical constants changed. The same motive is present in relativistic cosmology (our universe is one, but cosmological models are many!). The idea has also found some artistic expressions. In their excellent novel *The Compleat Enchanter*, L. Sprague de Camp and F. Pratt illustrate the meaning of our physical laws by letting their heroes visit a parallel universe where these laws are broken [38]. In the well known S/F story by George Gamow [39], an expedition of adventurers travels to a distant jungle where the Planck constant is greater than in the rest of the universe. What they face is a deformed version of our reality: The gazelles form interference patterns after running through the bamboo wood. The old elephants have their skins visibly fuzzy: The result of a slow dissolution of the wave packets. The entire story is naive (the single elephant does not need to look fuzzy, even if its mass center is uncertain.) Yet, can it tell us something?

As it seems, Gamow had some right intuitions, which can be formulated much better now [33, 34]. The $\hbar$-deformation is a natural paradigm to see the true magnitude of the Schrödinger problem (the *caricature* helps to see the reality!) Thus, e.g., consider a planetoid in cosmic space. When thinking about a cosmic body it is difficult to avoid a certain oversimplified picture: It includes the intrigues of the star-trek, but it does not include the Planck constant. Yet, even if small, this constant exists. If the quantum theory is universal, the mass center of the planetoid after a sufficiently long time should show some quantum uncertainty. The effect is extremely small, but it can be of fundamental importance. (As the matter of fact, the quantum diffusion of heavy blocks of metal was carefully taken into account as an obstacle to detect the gravitational waves [40]). Imagine now, that some beings live on the surface of our planetoid, and can observe the surrounding universe. What will they see? Since the planetoid center of mass is diffused, will the position of the distant stars be "fuzzy" in their eyes? Is the problem limited to a planetoid? Could one quantum object become a reference frame for other quantum objects?

The idea of *quantum frames* was put forward in 1984 by Aharonov and Kaufherr [41]; in a different form by Wódkiewicz [42]. Apart of its philosophical appeal, the problem cannot be avoided in several areas, including quantization of gravity. Yet, perhaps due to our macroscopic clumsiness, the idea has merely emerged... In a world with a huge Planck constant, it would already cause a 'wind of panic' (the space is crossed by *quantum planetoids*, the fundamental concept of the *inertial frame* is affected by the quantum deformation!). In our own universe, though, $\hbar$ is so small and the effects so tiny, that the creative panic still did not arrive...

While we are still waiting, let me report some little steps of control theory which can compensate the smallness of $\hbar$, bringing us closer to the moment of the confrontation.

## 2. Size deformations

Even if quantum mechanics holds for macroscopic objects, their quantum behavior (interference) may be masked by a number of effects making increasingly difficult the repetitions of Young's experiment:

1) de Broglie waves of heavy micro-objects are very short;

2) For heavy microscopic objects the wave fronts are narrow and diffract slowly; the wave packets resemble needle beams propagating along the classical trajectories [43].

A way to avoid the first difficulty could be to create beams of extremely slow microparticles (for charged particles this can be achieved by applying electric fields; for neutral ones, the Doppler cooling is available [44, 45]).

To avoid the second difficulty one can try to split the wave packet on an obstacle, but above $C_{60}$ the method might fail (the packet will be split but the coherence might be lost). Yet, for charged particles, the quantum control theory admits a class of operations which can widen the wave packets without destroying the coherence.

The phenomena of *squeezing/amplification* were originally discovered for the photon states in quantum optics [46]. Soon, it was shown that they can be repeated for charged massive particles. In the 1-dimensional non-relativistic quantum mechanics the simplest such effect is produced by the scale operator:

$$U = e^{i\mu\frac{qp+pq}{2}}, \qquad \mu \in \mathbf{R}, \tag{1}$$

transforming the Schrödinger wave packets according to:

$$\psi(x) \to (U\psi)(x) = \frac{1}{\sqrt{\sigma}}\psi\left(\frac{x}{\sigma}\right), \qquad \sigma = e^{\mu}. \tag{2}$$

An elementary technique to generate (1–2) as an evolution operation consists in applying time-dependent oscillator potentials; the Hamiltonian $H(t)$ and the evolution operator $U(t,t_0)$ are given by:

$$\begin{aligned} H(t) &= \frac{p^2}{2} + \gamma(t)\frac{q^2}{2} && (3)\\ \frac{dU}{dt} &= -iH(t)U(t,t_0) && (4) \end{aligned}$$

(we put $m = \hbar = 1$). Note, that (1–2) can be easily generated inside of the ion traps [47]. The operational programs for $\gamma(t)$ to produce (1–2) at an appointed time $t = \tau$ apply multiple methods, [46, 48, 49, 50, 54, 51, 52, 53], helped by the formalism of Milne (1926) [55] and by elements of the Sturm-Liouville theory. As a matter of fact, we shall show that one of the traditional Sturm-Liouville problems, when carefully read, gives all necessary information about the squeezing together with the numerical data needed to program the phenomenon. The simplest key is contained in the following *Helmholtz equation* [56]:

$$\frac{d^2\xi}{dx^2} + \lambda\phi(x)\xi(x) = 0. \tag{5}$$

The *eigenvalue problem* of Helmholtz consists in looking for the $\lambda$-values which permit non-trivial solutions of (5) vanishing on the extremes of a certain fixed interval $[a,b]$:

$$\xi(a) = \xi(b) = 0. \tag{6}$$

To see the meaning of (5–6) for the generation of the *squeezed states* it is enough to apply a sequence of simple reinterpretation steps.

*Step one*. We interpret (5) as a classical motion problem for a point particle on the Poincaré plane. Denote: $x = t$ (and call it "time"); $\xi = q$, $\xi' = p$; then (5) leads to the pair of canonical equations:

$$\begin{aligned} \frac{dq}{dt} &= p \\ \frac{dp}{dt} &= -\lambda\phi(t)q \end{aligned} \tag{7}$$

which describe a classical point particle obeying the Hamiltonian (3) with $\gamma(t) = \lambda\phi(t)$. Now, if $\phi(t) \geq 0$, $\phi \neq 0$ and $\lambda > 0$, the attractive oscillator in (7) transports the classical point $(q(t), p(t))$ around the phase space origin. Let the initial point be $q(a) = 0$, $p(a) \neq 0$, then as $\lambda$ tends to infinity, the end point $q(b)$, $p(b)$ must cross the $p$-axis an increasing number of times. Each time this happens, $\lambda$ is an eigenvalue of (1–2). Thus, the eigenproblem (5–6) has a discrete spectrum forming an infinite ladder of positive eigenvalues. However, the spectral problem (5–6) still contains more information.

*Step two*. We read the optical data. In the typical Sturm Liouville problem one looks only for the solution of (5) which fulfills the boundary condition (6). The second independent solution is forgotten as unphysical. Yet, this solution precisely brings an interesting message. Since (7) are linear and homogeneous, the solution $q(t)$, $p(t)$ for any $\lambda$ depends linearly on the initial values:

$$\begin{pmatrix} q(t) \\ p(t) \end{pmatrix} = u(t\,,\,a) \begin{pmatrix} q(a) \\ p(a) \end{pmatrix}, \tag{8}$$

where $u(t,a)$ is the symplectic *evolution matrix* for the time interval $[a,t]$. Consider now the matrix $u(b,a)$. Note that it must have a special form whenever $\lambda$ is one of the spectral values of (5–6). Indeed, the existence of a non-trivial $q(t)$ with $q(a) = q(b) = 0$ means that $u(b,a)$ transforms $(0, p(a))$ into $(0, p(b))$; henceforth, it must be of the form:

$$u(b,a) = \begin{pmatrix} \sigma & 0 \\ \eta & 1/\sigma \end{pmatrix}, \qquad \sigma, \eta \in \mathbf{R},\ \sigma \neq 0. \tag{9}$$

The matrix (9) gives now an information about the trajectories of (5) *which do not fulfill the boundary condition* (6). In fact, consider the congruence of classical orbits which depart at $t = a$ from a certain common initial point $(q_0, p)$, ($q_o$ fixed $\neq 0$, $p$ varies). If $\lambda$ is an eigenvalue of (5–6), then $u(b,a)$ is of form (9), and (8) implies:

$$\begin{pmatrix} q(b) \\ p(b) \end{pmatrix} = \begin{pmatrix} \sigma q_0 \\ \eta q_0 + \frac{p}{\sigma} \end{pmatrix}. \tag{10}$$

By plotting $q(t)$ against $t$, one sees that all orbits which diverge from a common point $q_0$ with varying $p$'s at $t = a$ must meet for $t = b$ at a common point $\sigma q_0$ with varying $p' = \eta q_0 + p/\sigma$. In optical terms, $\sigma q_0$ is *an image* of $q_0$ (squeezed if $|\sigma| < 1$, natural if $|\sigma| = 1$, amplified if $|\sigma| > 1$). The eigenvalues $\lambda$ of (5–6), are *the only real numbers* for which the congruencies obeying (7), departing from any common $q_0$ at $t = a$ must focus in a common point $\sigma q_0$ at $t = b$. The optical constant $\sigma$ is the squeezing (amplification) constant for the disregarded solutions of (5–6) [57].

*Step three*. We notice that the optical effect is not limited to the classical orbits. In fact, for the quadratic Hamiltonians, Equations (7) remain valid for the time-dependent canonical operators $q(t)$, $p(t)$, and the matrix (9) represents the dynamical evolution of the position and momentum observables in the Heisenberg frame. Thus, if $U(t,a)$ is the unitary evolution operation in $[a,t]$, if $q(a) = q$ and $p(a) = p$ are the initial canonical operators, and if $\lambda = \lambda_n$ is one of the eigenvalues $0 < \lambda_1 < \lambda_2 \ldots$, the evolution result at $t = b$ is:

$$U(b,a)^\dagger\, qU(b,a) = \sigma q \tag{11}$$

$$U(b,a)^\dagger pU(b,a) = \eta q + \sigma^{-1} p \tag{12}$$

($\eta, \sigma$ depending on $n$). Consistently, the evolution operator is [57]:

$$U(b,a) = P^n e^{i\ln|\sigma|\frac{qp+pq}{2}} e^{i\eta\frac{q^2}{2}} \tag{13}$$

($P$ is the parity operator) and transforms the wave packets according to:

$$\psi_U = [U(b,a)\psi] = \frac{1}{\sqrt{\sigma}} e^{i\eta\frac{x^2}{2\sigma^2}} \psi\left(\frac{x}{\sigma}\right) \tag{14}$$

producing the scale operation ("size deformation") corrected by an $x$-dependent phase factor. What is essential, the packets undergo the size deformation without losing coherence [57]. Interestingly, if the function $\phi(t)$ is symmetric in $[a,b]$, then the effect is trivial ($|\sigma| = 1$), but for $\phi$ asymmetric, in general, $|\sigma| \neq 1$. For $|\sigma| > 1$, the method can be used to amplify the narrow wave packets of heavy micro-objects to exhibit their quantum behavior.

*Step four*. The method now permits to use the magnetic operations in order to squeeze/amplify the states of charged micro-objects in more than one dimension. Indeed, consider a homogeneous, time-dependent magnetic field $\mathbf{B}(t) = B(t)\mathbf{n}$ of a cylindrical solenoid. The correspondence between the time-dependent magnetic fields and the oscillators of the form (3–4) on the plane orthogonal to the solenoid axis, makes possible to program the magnetic pulses which generate the scale transformation of the charged wave packet in the plane orthogonal to **n** (this can be achieved, in particular, by applying the sinusoidally oscillating magnetic fields $B(t) = B_0 + B_1 \sin\omega t$ of adequate amplitudes and frequency [57, 58]). If the effect is an expansion, then apart of producing the amplified images in terms of classical trajectories, it will also magnify the wave front of each single particle in the solenoid. In theory, maintaining properly oscillating magnetic fields, one can expand the wave fronts of arbitrarily heavy, charged point-like bodies to any desired size, bringing them into a controversial state of macroscopic quantum superpositions.

## 3. Time distortions

The possibility of analogous *time distortions* is due to the phenomenon of closed evolution processes (evolution loops). An obvious loop is produced by the harmonic oscillator (but brings no consequence for our subject). The possibility of more general loop phenomena was known already in 1970 [59]. The first evolution loop containing intervals of the free evolution (and thus, making possible the time deformations), was accidentally found in 1977 [60] as an exact solution of the Baker-Campbell-Hausdorff problem (BCH) for the product of 12 exponential terms:

$$e^{-i\frac{p^2}{2}} e^{-i\frac{q^2}{2}} \ldots e^{-i\frac{p^2}{2}} e^{-i\frac{q^2}{2}} \equiv 1, \tag{15}$$

where $\equiv$ denotes the operator proportionality up to a phase factor ($K \equiv M$ means $K = e^{i\kappa}M$, where $\kappa \in \mathbf{R}$). Physically, the sequence (15) stands for the 6 operations of the free evolution (in time lapses $\tau = 1$) interrupted by 6 kicks of the attractive oscillator potential. The identity (15) is an *echo phenomenon* for the non-spin states; it means that the Schrödinger's particle, affected by the entire sequence of 12 operations, returns to its initial state (whatever this state was).

As subsequently shown, the loop (15) is just a particular case of a large family of loop identities solving exactly the BCH problem for the generators of $SO(1,2)$. Most of them suggest technologies of the "time manipulation". The simplest case is the following exponential identity [61]:

$$e^{-i\gamma\frac{p^2}{2}} e^{-i\alpha\Gamma\frac{q^2}{2}} e^{-i\beta\frac{p^2}{2}} e^{-i\gamma\Gamma\frac{q^2}{2}} e^{-i\alpha\frac{p^2}{2}} e^{-i\beta\Gamma\frac{q^2}{2}} \equiv 1, \qquad \Gamma = \frac{\alpha+\beta+\gamma}{\alpha\beta\gamma}. \tag{16}$$

Taking $\gamma = -\tau$ one obtains the formula for the free evolution achieved in a "distorted time":

$$e^{-i\tau\frac{p^2}{2}} \equiv e^{-i\alpha\Gamma\frac{q^2}{2}} e^{-i\beta\frac{p^2}{2}} e^{+i\tau\Gamma\frac{q^2}{2}} e^{-i\alpha\frac{p^2}{2}} e^{-i\beta\Gamma\frac{q^2}{2}}. \tag{17}$$

If $\tau < 0$, (17) describes the retrospection operation (free evolution inversion) achieved in the laboratory time $T = \alpha + \beta$ at the cost of three sharp pulses of an attractive oscillator potential. If $0 < \tau \leq T$, the sequence (17) produces the free evolution slowing, achieved again in $T = \alpha + \beta$ with the help of two negative (repulsive) and one positive (attractive) pulses. If $\tau > \alpha + \beta$, the free evolution is accelerated by a program of two positive and one negative oscillator pulses. Note that the first case can be immediately translated into a program of homogeneous magnetic pulses in a cylindrical solenoid causing the 'retrospection' of the Schrödinger wave function on the plane orthogonal to the solenoid axis (the $z$-dependence still obeying the ordinary free evolution). However, it is the third case (time acceleration) which might turn pertinent for our problem.

A different method of altering the time was designed in 1990 by Aharonov et al. [62]. It consists in controlled evolution interrupted by a measurement. The existence of the macroscopic superposition in this method is *a priori* assumed and the process is dissipative: A part of the ensemble is lost but a part which survives recovers its pure state from an arbitrarily deep past. As it seems, a variant of the same method could be used as well to accelerate rather than to invert the evolution.

Meanwhile, the schemes of non dissipative (pure) magnetic operations causing the time distortions of all three types were further elaborated in 1994 [63]. One of the solutions consists in a sequence of 12 rectangular pulses of homogeneous magnetic fields of amplitudes

$$B_1, B_2, -B_2, -B_1, \; B_1, B_2, -B_2, -B_1, \; B_1, B_2, -B_2, -B_1$$

arriving in 3 packets, each one composed of 4 parallel pulses $B_1, B_2, -B_2, -B_1$, subsequently in $x, y$, and $z$ directions. The simplest scenario might involve 3 sources of homogeneous, mutually orthogonal magnetic fields (e.g., three electromagnets) activated subsequently. The time duration of each pulse of intensity $B_i$ is $\tau_i$ (no matter the direction); the sum $\tau = 6(\tau_1 + \tau_2)$ is the total operation time. As can be calculated, the output depends on the dimensionless parameters $\beta_i = eB_iT/2mc$, $\vartheta_i = \tau_i/T$, where $T = \tau_1 + \tau_2$ is the time duration of each pulse pair $B_1, B_2$. Only if

$$\beta_1 \tan(\vartheta_1 \beta_1) + \beta_2 \tan(\vartheta_2 \beta_2) = 0 \tag{18}$$

the dynamical operation upon the charged particle in $\mathbf{R}^3$ generated by the 12 rectangular magnetic pulses is of the free evolution type:

$$U(\tau, 0) \equiv e^{-i\tau' \frac{\mathbf{p}^2}{2}}, \tag{19}$$

but the parameter $\tau'$ does not coincide with the real operation time $\tau$. One has:

$$\tau' = \theta \tau, \tag{20}$$

where $\theta$ is the *deformation coefficient* [63]:

$$\theta = \frac{1}{3} + \frac{2}{3} \cos^2 \gamma_2 \frac{\tan^2 \gamma_2}{\gamma_1 \tan \gamma_1 - \gamma_2 \tan \gamma_2}, \quad \gamma_i = \beta_i \vartheta_i. \tag{21}$$

Similarly as before, our formula was obtained for a sequence of Hamiltonians neglecting the field retardation (an approximation which might be valid at the laboratory scale). The algebraic identities are elementary (an exact case of BCH) though not without a physical implication. When repeated persistently, the operations can deform in a coherent way the free evolution of a heavy charged molecule, so that it will become a laboratory equivalent of one of George Gamow elephants. If such objects indeed exist.

It is my pleasure to dedicate these thoughts to the memory of Moshé Flato, an exceptional personality, who can hardly be imitated in the present day mathematical physics.

## References

1. Dirac, P. A. M.: *Principles of Quantum Mechanics*, Clarendon, Oxford, 1958.
2. von Neumann, J.: *Mathematical foundations of quantum mechanics*, Princeton University Press, Princeton, 1955.
3. Primas, H.: *Chemistry, Quantum Mechanics and reductionism*, Springer Verlag, Berlin, 1983.
4. Beltrametti, E. B. and Casinelli, G.: The logic of Quantum Mechanics, in: G. C. Rota, (ed.), *Encyclopedia of Mathematics*, Addison Wesley, London, 1981.
5. Heisenberg, W.: Ueber den anschaulichen Inhalt der quantentheorischen Kinematik und Mechanik, *Z. Phys.* **43** (1927), 172–198.
6. Bohr, N.: On the notions of causality and complementarity, *Dialectica* **2** (1948), 312–319; *Essays 1958-1962 on atomic physics and human knowledge*, Wiley-Interscience, New York, 1963.
7. Jammer, M.: *The conceptual development of quantum mechanics*, McGraw-Hill, New York, 1966.
8. Orwell, G.: *Nineteen eighty-four*, Penguin Books, England, 1989.
9. Schrödinger, E.: Die gegenvärtige Situation in der Quantenmechanik, *Naturwissenschaften* **23** (1928), 807–812, 823–828, 844–849.
10. Selleri, F.: *Quantum paradoxes and physical reality*, Kluwer Acad. Publ., Dordrecht, 1990.
11. Moore, W.: *Schrödinger, life and thought*, Cambridge University Press, Cambridge, 1990.
12. Wigner, E. P.: The problem of measurement, *Am. J. Phys.* **31** (1963), 6–15.
13. Einstein, A., Podolsky, B., and Rosen, N.: Can quantum-mechanical description of physical reality be considered complete? *Phys. Rev.* **47** (1935), 777–780.
14. Bell, J. S.: *Speakable and unspeakable in quantum mechanics*, Cambridge University Press, Cambridge, 1987.
15. Ghirardi, G. C., Rimini, A., and Weber, T.: Unified dynamics for microscopic and macroscopic systems, *Phys. Rev. D* **34** (1986), 470–491.
16. Diósi, L.: Models for universal reduction of macroscopic quantum fluctuations, *Phys. Rev. A* **40** (1989), 1165–1174.
17. Ghirardi, G. C., Grassi, R., and Rimini, A.: Continuous-spontaneous-reduction model involving gravity, *Phys. Rev. A* **42** (1990), 1057–1064.
18. Penrose, R.: On gravity's role in quantum state reduction, *Gen. Rel. Grav.* **28** (1996), 581–600.
19. Penrose, R.: *Shadows of the mind; an approach to the missing science of consciousness*, Oxford Univerxity Press, Oxford, 1994.
20. Flato, M.: *Le pouvoir des mathématiques*, Hachette, Paris, 1990.
21. Raimond, J.-M., Brune, M., and Haroche, S.: Reversible decoherence of a mesoscopic superposition of field states, *Phys. Rev. Lett.* **79** (1997), 1964–1967.

22. Haroche, S.: Entanglement, decoherence and the quantum/classical boundary, *Phys. Today* **51**, (Mar 1998), 235–238.
23. Greenberger, D. M.: The neutron interferometers as a device for illustrating the strange behaviour of quantum systems, *Rev. Mod. Phys.* **55** (1983), 875–905.
24. Badurek, G., Rauch, H., and Tuppinger D.: Neutron interferometric double-resonance experiment, *Phys. Rev. A* **34** (1986), 2600–2608; Sumhammer, J., Rauch, H., and Tuppinger, D.: Stochastic and deterministic absorption in neutron interference experiments, *Phys. Rev. A* **36** (1987), 4447–4455.
25. Caldeira, O. A. and Legget, A. J.: Quantum Tunnelling in a Dissipative System, *Ann. Physics* **149**, 374–456.
26. Legget, A. J.: Quantum mechanics and realism on the macroscopic level, *Ann. N. Y. Acad. Sci.* **480** (1986), 21–24.
27. Schmiedmayer, J., Chapman, M., Ekstrom, C., Hammond, T., Wehinger, S., and D. Pritchard: Index of refraction of various gases for sodium matter waves, *Phys. Rev. Lett.* **74** (1995), 1043–1047.
28. Chapman, M., Ekstrom, C., Hammond, I., Schmiedmayer, J., Wehinger, S., and Pritchard, D.: Optics and Interferometry with Na Molecules, *Phys. Rev. Lett.* **74** (1995), 4783–4786.
29. Hegerfeld, G. C. and Kohler, T.: Atomic versus modular diffraction: influence of breakups and finite size, *Phys. Rev. A* **57** (1998), 2021–2029.
30. Snadden, M. J., McGuirk, J. M., Bouyer, P., Haritos, K., G., and Kasevich, M. A.: Measurement of the Earth gravity gradient with an atom interferometer-based gravity gradiometer, *Phys. Rev. Lett.* **81** (1998), 971–974.
31. Arndt, M., Nairz, O., Vos-Andrae, J., Keller, C., van der Zouw, G., and Zeillinger, A.: Wave-particle duality of $C_{60}$ molecules, *Nature* **401**, 14 October (1999), 680–682.
32. Moyal, J. E.: Quantum mechanics as a statistical theory, *Proc. Cambridge. Phil. Soc.* **45** (1949), 99–124.
33. Bayen, F., Flato, M., Fronsdal, C., Lichnerowicz, A., and Sternheimer, D.: Deformation theory and quantization I, II, *Ann. Physics*, **111** (1978), 61–151.
34. Kontsevich, K.: Formality conjecture, in: D. Sternheimer et al., (eds.), *Deformation Theory and symplectic Geometry*, Math. Phys. Stud. 20, Kluwer Acad. Publ., Dordrecht, 1997, pp. 139–151.
35. Fedosov, B. V.: Non Abelian reduction in deformation quantization, *Lett. Math. Phys.* **43** (1998), 137–154.
36. Tosiek, J. and Przanowski, M.: Weyl-Wigner-Moyal Formalism I. Operator Ordering, *Acta Phys. Polon. B* **26** (1995), 1703–1716; Plebański, J., Przanowski, M., and Tosiek, J.: The Weyl-Wigner-Moyal Formalism II. The Moyal Bracket, *Acta Phys. Polon. B* **27** (1996), 1961–1990; Przanowski, M. and Tosiek, J.: The Weyl-Wigner-Moyal formalism III. The generalized Moyal product in the curved phase space, *Acta Phys. Polon. B* **30** (1999), 179–201.
37. Kontsevich, M.: Operads and motives in deformation quantization, *Lett. Math. Phys.* **48** (1999), 35–72.
38. Sprague de Camp, L. and Pratt, F.: *The Compleat Enchanter*, Ballantine Books, New York, 1975.
39. Gamow, G.: *Mr. Tompkins in wonderland*, Cambridge University Press, Cambridge, 1953.
40. Caves, C. M., Thorne, K. S., Drever, R. W. P., Sandberg, V. D., and Zimmermann, M.: On the measurement of a weak classical force coupled to a quantum mechanical oscillator, *Rev. Mod. Phys.* **52** (1980), 341–392.
41. Aharonov, Y. and Kaufherr, T.: Quantum frames of reference, *Phys. Rev. D.* **30** (1984), 368–385.
42. Wódkiewicz, K.: Operational approach to phase-space measurements in quantum mechanics, *Phys. Rev. Lett.* **52** (1984), 1064–1067; On the operational uncertainty relation, *Phys. Lett. A* **124** (1987), 207–210.

43. Bagrov, V. G., Belov, V. V., and Ternov, I. M.: Quasiclassical trajectory-coherent states of a particle in an arbitrary electromagnetic field, *J. Math. Phys.* **24** (1983), 2855–2859.
44. Cohen-Tannoudji, C.: Manipulating atoms with photons *Rev. Mod. Phys.* **70** (1998), 707–719.
45. Phillips, W. D.: Laser cooling and trapping of neutral atoms, *Rev. Mod. Phys.* **70** (1998), 721–741.
46. Yuen, H.: Two-photon coherent states of the radiation field, *Phys. Rev. A* **39** (1976), 2226–2243.
47. Fernandez, C. D. J.: Transformations of a wave packet in a Penning trap, *Nuovo Cim. B* **107** (1992), 885–894.
48. Brown, L. S. and Carson, L. J.: Quantum-mechanical parametric amplification, *Phys. Rev. A* **20** (1979), 2486–2497.
49. Ma, S. and Rhodes, C.: Squeezing in harmonic oscillators with time dependent frequencies, *Phys. Rev. A* **39** (1988), 1941–1947.
50. Grübl, G.: Dynamical squeezing in quantum mechanics, *J. Phys. A* **22** (1989), 3243–3252.
51. Lewis Jr., H. R.: Classical and quantum systems with time-dependent harmonic-oscillator-type Hamiltonians, *Phys. Rev. Lett.* **18** (1967), 510–512; Class of exact invariants for classical and quantum time-dependent harmonic oscillators, *J. Math. Phys.* **9** (1968), 1976–1986.
52. Malkin, A., Man'ko, V. I., and Trifonov, D. A.: Coherent states and transition probabilities in a time-dependent electromagnetic fields, *Phys. Rev. D* **2** (1970), 1371–1385.
53. Ansari, N. A. and Man'ko, V. I.: Time-dependent squeezing and photon anti-bunching in squeezed even and odd coherent states, *Phys. Lett. A* **223** (1996), 31–36.
54. Ge, Y. C. and Child, M. S.: Nonadiabatic geometrical phase during cyclic evolution of a Gaussian wave packet, *Phys. Rev. Lett.* **78** (1997), 2507–2510.
55. Milne, W. E.: The numerical determination of characteristic numbers, *Phys. Rev.* **35** (1930), 863–867.
56. Bender, C. M. and Orzag, S. A.: *Advanced Mathematical Methods for Scientists and Engineers*, McGraw-Hill, New York, 1978.
57. Delgado, C. F., Mielnik, B., and Reyes, M. A.: Squeezed states and Helmholtz spectra, *Phys. Lett. A* **237** (1998), 359–364.
58. Delgado, C. F. and Mielnik, B.: Magnetic control of squeezing effects, *J. Phys. A* **31** (1998), 309–320.
59. Malkin, A. and Man'ko, V. I.: Coherent states and the Green function of a charged particle in varying electric and magnetic fields, *JETP* **59** (1970), 1746–1754.
60. Mielnik, B.: Global mobility of Schrödinger's particle, *Rep. Math. Phys.* **12** (1977), 331–339.
61. Mielnik, B.: Evolution loops, *J. Math. Phys.* **27** (1986), 2290-2306.
62. Aharonov, Y., Anandan, J., Popescu, S., and Vaidman, L.: Superpositions of time evolutions of a quantum system and a quantum time-translation machine, *Phys. Rev. Lett.* **64** (1990), 2965–2968.
63. Fernandez, C. D. J. and Mielnik, B.: Controlling quantum motion, *J. Math. Phys.* **35** (1994), 2083–2104.