Singletons and neutrinos

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Abstract. The first half is a rapid review of 30 years of work on physics in anti-de Sitter space, with heavy emphasis on singletons. Principal topics are the kinematical basis for regarding singletons as the constituents of massless particles, and the effect of (negative) curvature in the infrared domain. Ideas that lead to an alternative to Big Bang cosmology are merely sketched. The second half presents new ideas inspired by experimental results on neutrino oscillations. Since leptons are massless before symmetry breaking it is natural to view them as composite states consisting of one Bose singleton (the Rac) and one Fermi singleton (the Di). This gives rise to a particular formulation of the phenomenology of electroweak physics, and strong suggestions for an expansion of the Standard Model. An expansion of the Higgs sector seems inevitable, and flavor changing symmetry, complete with a new set of heavy vector mesons, is a very attractive possibility.

Keywords: singleton, neutrino, flavor symmetry

Mathematics Subject Classifications (2000): 81T10, 81T20

Dedicated to the memory of my friend Moshé Flato

1. Introduction

This talk is intended as a modest offering to the memory of Moshé Flato, the most original physicist that I have known, and my best friend for more than 25 years.

Perhaps you will say that Moshé was a mathematician as much as he was a physicist. Indeed, he was a professor of mathematics and he created, from nothing, one of the finest departments of mathematical physics anywhere; nevertheless, his great passion was physics. Others will tell you about some of Moshé's greatest accomplishments in mathematics and in mathematical physics; it falls on me to summarize his work on the **physics** of spaces of constant curvature, an endeavor that is entirely in the realm of physics.

As it happens, this work, on which I shall report, was done in collaboration with others, including Daniel Sternheimer and myself, and the early part of it was initiated before we started to publish together, but Moshé's influence dates back to the beginning.

2. Physics in anti-de Sitter space

The initial problematic was very simple: The cosmological constant is small, but it can never be shown, experimentally, to be exactly zero. Any value other than zero is incompatible with the idea that the Poincaré group is the basic symmetry

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group of space time, but it is consistent with a symmetry group that has the same dimension as the Poincaré group; all of the most important concepts of flat space physics can be extended to a space of constant curvature that admits the global action of one of the two de Sitter groups, $SO(3,2)$ or $SO(4,1)$.

This idea was not new in the mid-sixties (see e.g. [34, 21, 25]). But up to that time, of the two possibilities, $SO(4,1)$ had received the most attention although it was known, already, that symmetry under this group leads unavoidably to spontaneous creation of matter (see e.g. [26]).

Both of us had a great admiration for Wigner's work [33] on the unitary representations of the Poincaré group that led to the first classification of elementary particles, in terms of mass and spin (or helicity). It was very natural to apply the same philosophy to the classification of elementary particles in (anti-) de Sitter space. The unitary representations of both groups were essentially known at the time (cf. [32, 8] for $SO(4,1)$ and [9] for the discrete series of $SO(3,2)$); the full unitary dual was calculated only at the end of the seventies [1]), but the interpretation in terms of elementary particles was not yet well developed.

When the problem is approached in this way, then immediately one is led to favor $SO(3,2)$, the symmetry group of anti-de Sitter space (so called later on, and now abbreviated as AdS), over the other alternative. This is because $SO(3,2)$ has representations that can be associated with elementary particles, while $SO(4,1)$ does not. The energy spectrum of every unitary representation of $SO(4,1)$, except the trivial one, is the real line, unbounded in both directions [30], and if there is one physical principle that has stood the test of time, it is the requirement that energy must be bounded below. Consequently, all our work was concerned exclusively with $SO(3,2)$ and anti-de Sitter space time.

The choice of $SO(3,2)$ implies a negative curvature. This curvature, in the cosmological context, is very small, and it was never expected to be measurable. Our project was not concerned with the *magnitude* of the cosmological constant, but merely with the fact that, as a matter of principle, it *need not be zero*. It was expected that physics in a space of constant, negative curvature was possible and above all that its elaboration would be very educational. That it would actually turn out to suggest new types of physical phenomena was a great surprise.

The first indication of new physics appeared in connection with "massless" particles. It turns out that "masslessness" is a term that can be applied to certain irreducible representations of $SO(3,2)$ with as much justification as in the context of the Poincaré group [3]. One instance of masslessness appears in anti-de Sitter electrodynamics [15].

For the sake of simplicity, I shall not deal directly with realistic Maxwell theory, but instead with the theory of a spinless, massless field. All the interesting features of a gauge theory are then lost, but the features that I want to discuss are not. (The realistic picture involves Gupta-Bleuler triplets and indecomposable representations [15, 12].) In flat space free (spinless) photons are associated with an irreducible representation of the Poincaré group; this particular representation has the interesting property of having a unique extension to the conformal group [2]. The conformal group in anti-de Sitter space time is the same as the conformal group in flat space, locally isomorphic to $SO(4,2)$, and the same irreducible representation of the conformal group appears in (scalar) electrodynamics, in both cases. But in the case of anti-de Sitter space this irreducible representation breaks up into two inequivalent representations of $SO(3,2)$. This means that there are two kinds of photons, with different propagators. In order to incorporate conformal invariance into the theory of free quantum fields, one must quantize the field in such a way that both types propagate. The big surprise is that this is incompatible with the self-adjointness of the Hamiltonian. In other words, energy is not conserved. To conserve energy, one must use only one of the two types of photons; this amounts to spontaneous breakdown of conformal symmetry.

Now I should like to present here for the first time an idea that would have been elaborated by Moshé and myself if Moshé had been given more time. It is a radical idea, for it suggests an alternative to the Big Bang, and thus it is likely to irritate some people; but I will risk it. The idea is to accept the lack of energy conservation that is implied by conformal invariance of QED. This will lead to spontaneous creation of energy, locally throughout the universe. Conceivably, the amount of energy (and matter) created could balance the loss occasioned by the divergence of matter in the form of visible galaxies, and lead to a kind of "steady state" model of cosmology, characterized by a mean mass density that is constant in time. Note that thermal equilibrium would not be reached, and some of the ideas [29] that attempt to explain the preponderance of baryons would become more viable. Some people (including Fred Hoyle [22]) are more comfortable with this scenario.

Before we turn to the more spectacular aspects of anti-de Sitter physics, it is worth while to stress, one more time, the theoretical benefits of negative curvature, however small. The energy spectra of elementary particles in anti-de Sitter space are positive definite; thus there are no infrared singularities. Given the pivotal role of the infrared catastrophe in QCD, I marvel at the fact that no attempt has yet been made to investigate the effect of negative curvature on the confinement problem.

3. Singletons

By far, the most dramatic consequence of allowing for a small negative curvature is the existence of singletons. These are highly degenerate representations of $SO(3,2)$, with positive energy and thus at first sight associated with elementary particles. They were discovered by Dirac in 1963 [7]. As elementary particles they were at first dismissed by us on the grounds that these representations have too few states to allow for the formation of localized wave packets. (One manifestation of this is the singleton black body spectrum: it turns out to be that of ordinary particles in 3-dimensional space-time [15].) But this property is precisely what gives singletons their fascinating properties [11]. A free singleton with fixed energy has a well defined angular momentum. If the energy is large enough to be measurable then the angular momentum is enormous and the state may be associated with sloshing modes of the universe; if the angular momentum is small then the energy is of the order of the curvature and thus too small to be observed.

Our first important observation was that all 2-singleton states are massless; in fact the action of $SO(3,2)$ on the space of 2-singleton states breaks up into an infinite direct sum of massless representations. For example, if $\varphi(x)$ creates a singleton, then $\varphi(x)\varphi(x)$ creates a massless particle with spin zero, and $\varphi(x)\partial_\mu\varphi(x)$ creates a photon. This property of singletons, for which there is no analog in flat space, suggests a model of massless particles as 2-singleton composites. No interaction and no binding energy is associated with this type of compositeness; it is just a kinematical fact. Composite electrodynamics was presented in [12] and the linear approximation to composite gravity in [13].

4. Singletons and electroweak interactions

Rather than continuing with this review of work that has been published (references may be found in our last paper with Moshé [14]) I prefer to present some new ideas. Moshé had a very strong belief in the physical role of singletons. As a tribute to Moshé, Daniel Sternheimer and I have done our best to vindicate this idea – feeling that it is something we owe our friend.

The Standard Model is based on "the weak group", $S_W = SU(2)\otimes U(1)$, and more precisely on the Glashow representation of this group, that is carried by the triplet $(\nu_e, e_L; e_R)$ and by each of the other generations of leptons. Let us now suppose that

(a) There are three bosonic singletons $(R^N R^L; R^R) = (R^A)_{A=N,L,R}$ (three "Rac"s) that carry the Glashow representation of S_W;

(b) There are three spinorial singletons $(D_\varepsilon, D_\mu; D_\tau) = (D_\alpha)_{\alpha=\varepsilon,\mu,\tau}$ (three "Di"s). They are insensitive to S_W but transform as a Glashow triplet with respect to another group S_F (the "flavor group"), isomorphic to S_W; (c) The vector mesons of the standard model are Rac-Rac composites, the leptons are Di-Rac composites, and there is a set of vector mesons that are Di-Di composites and that play exactly the same role for S_F as the weak vector bosons do for S_W: $W_A^B = \bar{R}^B R_A$, $L_\beta^A = R^A D_\beta$, $F_\beta^\alpha = \bar{D}_\beta D^\alpha$.

The vector mesons are associated with conserved currents and fall into a category of composite particles that was described in Moshé's last paper [13]. There is not any strong evidence, at this time, that S_F is isomorphic to S_W, only that S_F has a representation of dimension 3. The assignments of transformation properties of Di's and Rac's can be interchanged.

We propose a slightly more economical model; namely we shall identify the two $U(1)$s with each other. There is only one $U(1)$; the symmetry group is

$SU(2)_W \otimes U(1) \otimes SU(2)_F$. The subgroup $SU(2)_W$ acts on the Racs, $SU(2)_F$ acts on the Dis, and the hypercharge generator of $U(1)$ acts on both.

Let us concentrate on the leptons ($A = N, L, R$; $\beta = \varepsilon, \mu, \tau$)

$$(L^A_\beta) = \begin{pmatrix} \nu_e & e_L & e_R \\ \nu_\mu & \mu_L & \mu_R \\ \nu_\tau & \tau_L & \tau_R \end{pmatrix} . \tag{1}$$

It is a remarkable fact that there are very good reasons to believe that this collection of leptons, precisely three complete generations, is complete. If leptons are composite, and if lepton fields are related to bilinears, then the constituents must include both bosons and fermions and the factorization $L^A_\beta = R^A D_\beta$ is strongly urged upon us by the nature of the phenomenological summary in Equation 1.

Fields in the first two columns couple horizontally to make the standard electroweak current, those in the last two pair off to make Dirac mass-terms. Particles in the first two rows combine to make the (neutral) flavor current and couple to the flavor vector mesons.

The Higgs fields have a Yukawa coupling to lepton currents,

$$\mathscr{L}_{\rm Yu} = -g_{\rm Yu} \bar{L}^\beta_A L^B_\alpha H^{\alpha A}_{\beta B}. \tag{2}$$

The Standard Model was constructed with a single generation in mind, hence it assumes a single Higgs doublet, and must therefore introduce three independent Yukawa coupling constants, $g_{\rm Yu} H^{\alpha A}_{\beta B} \to \delta^\alpha_\beta g_\alpha H^A_B$.

However, an early and remarkable property of weak interaction phenomenology was electron-muon universality. In a theory based on intermediary vector mesons, this is expressed as an equality of the coupling constants of the basic interaction lepton-lepton-meson. In Weinberg-Salam theory, with its spontaneous breakdown of Yang-Mills symmetry, this equality of coupling constants is natural and of geometric origin. If, as has been proposed, the Higgs field also has a geometrical meaning [6, 5], then it is natural to suppose that the Yukawa couplings lepton-lepton-Higgs are also characterized by a universal coupling constant; the same for electron, muon and (?) tau. This symmetry between leptons is broken only spontaneously, by the spread of the vacuum expectation values (VeVs) of the Higgs field.

Symmetry with respect to the group $SU(2)_W \otimes U(1) \otimes SU(2)_F$ does not justify Equation 2 with a single Yukawa coupling constant. We shall return to this point below. It is assumed that all the neutrinos are lefthanded.

If the great number of components of Higgs fields is unwelcome, it should be kept in mind that the usual Higgs field is not widely believed to correspond to an elementary particle. If it is composite, then there is no reason to expect it to have only two components. In our model, where leptons and vector mesons are composed of singletons, it would be natural to suppose that the Higgs field is likewise composed of singletons, and the topological structure of singleton field theory even suggests that there may be no elementary particles associated with the Higgs field in the full dynamical theory of the future.

Nonvanishing vacuum expectation values of the neutral components are directly related to charged lepton masses,

$$g_{\text{Yu}}\langle H^{\alpha L}_{\alpha R}\rangle = m_\alpha,\ \alpha = \varepsilon, \mu, \tau. \tag{3}$$

The coupling constants in m_ε and m_μ must be equal, but the third one, associated with the tau, can be different. If these are the only components with nonvanishing VeV, then the following masses are induced for the weak vector mesons:

$$m^2(W^\pm) = g^2\sum_\alpha \langle H^{\alpha L}_{\alpha R}\rangle^2, \quad m^2(Z) = (g^2+g'^2)\sum_\alpha \langle H^{\alpha L}_{\alpha R}\rangle^2, \tag{4}$$

where g and g' are the two fundamental coupling constants of the Standard Model ($\tan\theta_W = g'/g$), and

$$m^2(C^\pm) = h^2(\langle H^{\mu L}_{\mu R}\rangle^2 - \langle H^{\varepsilon L}_{\varepsilon R}\rangle^2), \tag{5}$$

where h is the gauge coupling constant associated with the flavor group $SU(2)_F$. The theory is still invariant under the three abelian groups associated with the three lepton numbers, but only $L_\mu - L_\varepsilon$ is gauged, by the flavor gauge boson C_3 that remains massless, so far. The hypercharges of the leptons are the same as in the Standard Model and the hypercharges of the Higgs field are determined by the postulated invariance of the interaction.

5. The new developments

Neutrino oscillations, especially between the two neutrinos associated to the muon and to the tau, appear to have been firmly established [31, 4, 27, 10, 28]. This suggests non-vanishing neutrino masses but does not imply it, especially if new flavor changing interactions are not ruled out [20, 19, 23]. Nevertheless, there are several attractive mechanisms that account for most of the data and give masses to at least some of the neutrinos, without introducing any additional leptons. The most economical assumption is that the masses of ν_μ and ν_τ are of the order of 0.1 eV and that of ν_e even smaller, possibly zero. It was pointed out, almost 20 years ago [17, 16, 24], that there is room for an additional Higgs field, an $SU(2)$ triplet, and that this implies non-vanishing neutrino masses. Our proposal is slightly different.

We postulate additional Higgs fields, coupled to leptons in the following way,

$$\mathscr{L}'_{\text{Yu}} = h_{\text{Yu}} L^A_\alpha L^B_\beta K^{\alpha\beta}_{AB} + \text{h.c.}. \tag{6}$$

As in the case of Equation 3, this formula should not be taken quite literally, for symmetry does not require that all the couplings have the same strength.

We now give a nonvanishing vacuum expectation values to the neutral components (generalizing [GR])

$$\langle K^{\alpha\beta}_{NN}\rangle,\ \alpha,\beta = \varepsilon, \mu, \tau. \tag{7}$$

This implies a general, symmetric neutrino mass matrix. In addition, one of the two remaining massless vector mesons becomes massive. The masses are

$$\begin{aligned}
m^2(W^\pm) &= g^2\Big(\sum_\alpha \langle H^{\alpha L}_{\alpha R}\rangle^2 + 2\sum_\alpha \langle K^{\alpha\tau}_{NN}\rangle^2\Big),\\
m^2(C^\pm) &= 2h^2\big(\langle H^{\mu L}_{\mu R}\rangle - H^{\varepsilon L}_{\varepsilon R}\rangle\big)^2,\\
m^2(C^3) &= h^2\sum_\alpha \langle K^{\alpha\tau}_{NN}\rangle^2,\\
m^2(Z) &= (g^2+g'^2)\Big(\sum \langle H^{\alpha L}_{\alpha R}\rangle^2 + 4\sum_\alpha \langle K^{\alpha\tau}_{NN}\rangle^2\Big).
\end{aligned}$$

6. Other predictions of the model

The interpretation of the atmospheric neutrino experiments must take into account, not only neutrino masses and mixing, but also the effect of the flavor-changing interactions induced by the new vector mesons. To account for the observed smallness of flavor changing interactions it is necessary that the new VeV be at least of the order of magnitude of 100 GeV, so that h_{Yu} must be very small. Strong constraints on h_{Yu}, and h are imposed by past and future experiments involving neutral currents. As these become more accurate the model may fail.

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