Titles and Abstracts

Scientific Conference
Advances and Prospects in Physical Mathematics

Yakir Aharonov, Tel Aviv & South Carolina

A new approach to quantum mechanics

A reformulation of quantum mechanics using two vectors in Hilbert space (instead of one) to describe the state of a quantum system at a given time is presented. This reformulation suggests new effects and new features of quantum mechenics and opens a way to a generalization of quantum mechanics that will resolve the longstanding problem of the collapse of the wave function.

Louis Boutet de Monvel, Paris VI

Star Produits Réels et Complexes – Real and Complex Star Products

Dans cette conférence je voudrais donner un aperçu des star-produits et de la contribution de Moshé Flato à ce sujet. Dans la deuxième partie de la conférence je rappelerai la définition que j'utilise, qui est un peu plus générale pour englober le calcul pseudo-différentiel, et indiquerai les problèmes qui se posent dans l'étude des star-produits holomorphes.

In this talk I wish to give a brief survey on star products and of Moshé Flato's contribution to the subject. In the second part of the talk I will recall the definition I use, which is slightly more general so as to include the pseudo-differential calculus, and I will give indications on the problems that arise for holomorphic star-products.

Alain Connes, Collège de France

Cyclic cohomology and Hopf algebras

Ludwig Faddeev, St.Petersburg & Helsinki

The modular double of a quantum group

There are many indications both in mathematics and in physical applications, that the deformed structure with parameter $q = e^{i\pi\tau}$ must be combined with one with $\tilde{q} = e^{i\pi(-1/\tau)}$. An explicit construction is presented on the example of the $\mathrm{SU}(2)_q$ quantum group with straightforward generalization for the $\mathrm{SU}(N)_q$ case. The main result is a formula for the universal R-matrix for the combined object, which we propose to call the "modular double". Factors of type II_1 appear naturally and a generalization of the Gel'fand-Kirillov theorem is discussed.

Boris Fedosov, Moscow & Potsdam

On G-trace and G-index in deformation quantization

The letter G in the title means a compact group acting by symplectomorphisms on a symplectic manifold M. There is a canonical G-invariant deformation quantization on M.

Consider an element s of G and a connected component of its fixed-point set. Assuming that it is a smooth compact submanifold of M and the action of s on its normal bundle is non-degenerate, we construct an s-trace functional on the deformed algebra of functions on M with support on the given connected component postulating the properties of the s-trace by analogy with the operator trace $\mathrm{Tr}SA$. Further, we define the s-index as the s-trace of the unit function and prove the index formula in the spirit of the equivariant formula of Atiyah-Segal-Singer.

SERGIO FERRARA, CERN & UCLA

Superconformal Field Theories, Multiplet Shortening, and the $AdS_5/SCFT_4$ Correspondence

We review the unitarity bounds and the multiplet shortening of UIR's of 4 dimensional superconformal algebras SU(2,2—N), (N=1,2,4) in view of their dual role in the AdS/SCFT correspondence. Some applications to KK spectra, non-perturbative states and stringy states are given.

JÜRG FRÖHLICH, ETHZ

Classical limits of Bose systems

CHRISTIAN FRØNSDAL, U.C.L.A.

Singletons and Neutrinos

The first half is a rapid review of 30 years of work on physics in anti-De Sitter space, with heavy emphasis on singletons. Principal topics are the kinematical basis for regarding singletons as the constituents of massless particles, and the effect of (negative) curvature in the infrared domain. Ideas that lead to an alternative to Big Bang cosmology are merely sketched. The second half presents new ideas inspired by experimental results on neutrino oscillations. Since leptons are massless before symmetry breaking it is natural to view them as composite states consisting of one Bose singleton (the Rac) and one Fermi singleton (the Di). This gives rise to a particular formulation of the phenomenology of electroweak physics, and strong suggestions for an expansion of the Standard model. An expansion of the Higgs sector seems inevitable, and flavor changing symmetry, complete with a new set of heavy vector mesons, is a very attractive possibility.

SIMONE GUTT, ULB & METZ

Variations on Deformation Quantization

This will be a promenade around some aspects of deformation quantization:

- Covariant star products and Kontsevich star product on the dual $\mathfrak{g}^*$ of a Lie algebra; an elementary proof of the equivalence of two universal star products on $\mathfrak{g}^*$.
- A Čech presentation of Deligne's class and equivalence of star products on a symplectic manifold.
- Convergence of star products; compact and noncompact Hermitian symmetric spaces.

Maxim Kontsevich, IHES

Geometry of the moduli space of non-commutative algebras

Bertram Kostant, M.I.T.

A Cubic Dirac Operator and a Generalization of the Bott-Borel-Weil Theorem

The Bott-Borel-Weil theorem (BBW) is a statement about a complex homogeneous space $X = G/R$ where G is a compact semisimple Lie group and R is the centralizer of a torus in G. One knows that BBW is equivalent to the determination of how R operates on the cohomology of a certain nilpotent Lie algebra of antiholomorphic tangent vectors. Upon replacing the complex structure with a space S of spinors BBW is equivalent to a statement about the eigenvalues of the Casimir operator of R in the module $S \otimes V_\lambda$ where V_λ is the irreducible G-module with highest weight λ. But then the complex structure is eliminated and the problem makes sense for any subgroup $R \subset G$ having the same rank as G. We solve the problem in this generality. Involved is a new (as least to mathematicians) Dirac operator with a cubic term whose square is expressed in terms of the Casimir operator. A key feature is that, associated to λ, is a multiplet of representations of R, having rather striking properties, and cardinality equal to the Euler number of X.

Elliott Lieb, Princeton

Stability of matter and the self-energy of an electron in non-perturbative quantum electrodynamics

Despite its phenomenal successes in calculating scattering cross sections and level shifts using perturbation theory, Quantum Electrodynamics (QED) has serious fundamental difficulties. Inquiries into the nature of bulk matter interacting with the radiation field, e.g., its stability, seem to be outside the scope of perturbation theory.

A possible approach to the stability of matter problem in the realm of QED below energy scales of the order of several electron rest masses will be outlined and one or two very modest steps will be shown.

QED does not make sense without the introduction of large energy cutoffs; infinities will invariably appear otherwise. Renormalization theory is based on the idea that the divergent terms can be absorbed into a few physical constants such as the mass and the charge of the electron and that what is left over as an effective theory is Schrödinger's equation with tiny QED corrections.

The first step in this program (joint work with *Michael Loss*) is to obtain precise estimates on how self-energies depend on the cutoffs when the mass is fixed. The simplest case of a nonrelativistic electron interacting with the radiation field already leads to challenging problems in analysis and a variety of results will be presented. The two main differences between our results and the usual ones is a) our results are non-perturbative and b) we explicitly discuss the dependence of the energy on N, the number of electrons, and with the help of the Pauli exclusion principle we obtain bounds proportional to N. Bosons, on the other hand, are definitely unstable because the energy is strictly sublinear in N, as we prove.

We treat various examples of the electron self-energy Hamiltonians besides the nonrelativistic one. E.g., the Pauli Hamiltonian (which includes the spin-field energy $\sigma \cdot B$), and the "relativistic" $\sqrt{(p+\sqrt{\alpha}A)^2+m^2}$. The nonrelativistic and the Pauli examples show that perturbation theory is definitely misleading in the sense that it predicts the wrong power law for the dependence of the self-energy on the cutoff; the self-energy is definitely not analytic in the fine structure constant, α, at $\alpha = 0$. This, we believe, is perhaps the first explicit discrepancy between perturbation theory and actual non-perturbative calculations in realistic models.

Wilfried Schmid, Harvard

L-functions attached to Maass modular forms

Let G be a reductive group defined over $\mathbb{Q}$, and Γ an arithmetic subgroup of $G_{\mathbb{R}}$, the group of real points. Every discrete summand of $L^2(\Gamma\backslash G_{\mathbb{R}})$ determines, and is determined by, some Γ-invariant distribution on the quotient of $G_{\mathbb{R}}$ modulo a parabolic subgroup. When $G_{\mathbb{R}}$ has real rank one, and conjecturally also in the more general case, the degree of regularity of such "automorphic distributions" can be described precisely. These regularity results lead to a new derivation of the functional equation for L-functions attached to Maass modular forms. Unlike Maass' approach to the functional equation, the new approach applies also to groups other than $Sl(2,\mathbb{R})$.

Jacques C.H. Simon, Dijon

Nonlinear relativistic evolution equations: survey of a new approach

This talk addresses to a large audience. It sums up the results obtained during the last years by Moshé Flato, Erik Taflin and myself in the field of relativistically covariant field equations, without entering into technicalities such as those of nonlinear group representations, related cohomologies or very subtle analysis. Two examples are used as support: the non-linear Klein-Gordon equation and the coupled Maxwell-Dirac equations. Scattering for small initial data is described. For Maxwell-Dirac equations, the solution to the infrared divergencies problem is presented.

Leon Takhtajan, SUNY Stony Brook

Quantum field theories on an algebraic curve

We formulate local and global quantum field theories on an algebraic curve over an arbitrary field of constants and discuss applications to the reciprocity laws and the class field theory.

Special Session
Sciences applied to insurance and finance
Topical contributions will include

HÉLYETTE GEMAN
Professor of Finance, University Paris IX Dauphine & ESSEC
From Bachelier and Lundberg to Insurance Derivatives

The talk will try to show the convergence of the fields of insurance and finance throughout the twentieth century. From a theoretical standpoint, both subjects – and particularly the latter one since actuarial sciences already existed in the 19th century – have benefited from the seminal work by Bachelier, Lundberg, Markov, Lv́y, Ito and many other great names. These results have led to continuous-time stochastic modelling of asset prices or loss processes in a random economy, the foundation of pricing and risk management. From an operational standpoint, the finance and insurance industry offer today similar products, whether they are investment vehicles such as life insurance contracts, or hedging instruments such as credit-linked notes, catastrophe bonds or insurance derivatives.

JEAN-CLAUDE SIMON
PDG, A2iA & professeur honoraire, Université P. & M. Curie
La Reconnaissance Automatique de l'Écriture Manuscrite

Il y a 15 ans la Reconnaissance Automatique de l'écriture manuscrite se limitait à des essais de laboratoire, à la limite de la Science Fiction. Notre recherche a permis de développer un logiciel qui est aujourd'hui installé dans 40 sites de production et reconnaît journellement les montants de 20% des chèques manuscrits français, non seulement en chiffres mais aussi en lettres. Des concepts scientifiques très nouveaux ont été dégagés par cette recherche. Il a fallu résoudre d'innombrables problèmes scientifiques et techniques, et pour cela développer dans des conditions difficiles une entreprise en France et à l'Étranger. Une démonstration détaillée de reconnaissance de montants de chèques manuscrits conclura l'exposé.