Noncommutative Geometry and Higher Structures

6-10 June 2022, Scalea, Italy

Table 1. Tentative Schedule. Talks last 30 minutes plus 10 for questions.

	Mon	Tue	Wed	Thu	Fri
9.15 - 9.45	-	Chemla	Blohmann	Xu	Kowalzig
10.00 - 10.30	Waldmann	Laurent-Gengoux	Ronchi	Jotz-Lean	D'Andrea
10.45 - 11.15	Coffee Break				
11.15 - 11.45	Gavarini	Cattaneo	Giaquinto	Tortorella	-
12.00 - 12.30	Dippell	Lunch 12.30	Saracco	Santi	-
13.00	Lunch				
4 - 6	Free Discussion				
6.00 - 6.30	Bekaert	-	Batakidis	Ryvkin	-
6.45 - 7.15	Schnitzer	-	Stiénon	Cattafi	-
7.30	Dinner				

Talk Data

Panagiotis Batakidis, Aristotle University Thessaloniki

Poisson cohomology of some singular Poisson structures in dimension 4

Abstract: The talk concerns two classes of singular Poisson structures in dimension 4. The ambient objects on which these structures live are broken Lefschetz fibrations (bLfs) and near-symplectic manifolds respectively. There is a close relationship between these two objects explained by Auroux, Donaldson & Katzarkov generalizing results of Donaldson & Gompf on the relationship between Lefschetz fibrations and symplectic manifolds. The Poisson structure on bLfs was constructed by García-Naranjo, Suárez-Serrato & Vera. We construct a Poisson structure on near-symplectic manifolds (around the singular locus) and compute the Poisson cohomology of both structures showing that the number and type of singularities in each case can be traced into these groups. This is joint work with R. Vera. □

Xavier Bekaert, Institute Denis Poisson

Universal enveloping algebras of Lie-Rinehart algebras: connections as crossed products

Abstract: It is well-known that a flat (or curved) connections on a transitive Lie algebroid is equivalent to a decomposition of the Lie-Rinehart algebra of its sections as a (curved) semidirect sum. We show that, analogously, the universal enveloping algebra of the latter Lie-Rinehart algebra factorises as a smash (or crossed) product. A corollary is that a flat (or curved) invariant Ehresmann connection on a principal bundle provides a factorisation of the associative algebra of invariant differential operators on the total space as a smash (or crossed) product of the associative algebras respectively spanned by invariant differential operators tangential to the fibre and by differential operators on the base manifold. □

Christian Blohmann, Max Plank Institut für Mathematik, Bonn
The homotopy momentum map of general relativity

Abstract: In general relativity, Noether's theorem does not give rise to a hamiltonian momentum map, which is a long-standing, fundamental, and annoying problem. Working in the setting of multisymplectic geometry, I will explain that there is a natural homotopy momentum map of general relativity, i.e. a morphism of L_∞ algebras that extends the map from spacetime vector fields to their conserved Noether currents. □

Francesco Cattafi, Universität Würzburg
Pfaffian Morita equivalence and geometric structures

Abstract: After revising the classical notion of Morita equivalence between Lie groupoids, I will outline how to "decorate" it with compatible geometric structures, inspired by the definition of symplectic Morita equivalence between symplectic groupoids, In particular, I will introduce a notion of "Pfaffian Morita equivalence" between Lie groupoids endowed with a special object originating from the theory of Lie pseudogroups. I will then explain how these "Pfaffian" notions are related with several well-known geometric structures on manifolds, and discuss the interplay of their properties with Morita equivalence, This is joint work with Luca Accornero. □

Alberto Cattaneo, Universität Zürich
Poisson structures from corners of field theories

Abstract: The BV formalism and its shifted versions in field theory have a nice compatibility with boundary structures. Namely, one such structure in the bulk induces a shifted (possibly degenerated) version on its boundary. I will discuss in particular how to proceed from the BFV structure on a "space" slice in field theory, which describes the symplectic reduction due to constraints), to a shifted structure on its boundary (the corners of space–time), which in turn describes a Poisson algebra (possibly up to homotopy). I will describe a few examples, including, time permitting, general relativity in the coframe formulation. □

Sophie Chemla, Université Sorbonne
Differential calculus over double Lie algebroids

Abstract: The theory of Lie algebroids allows to see in a unified way de Rham cohomology, Cartan Eilenberg Cohomology and Poisson-Lichnerowicz cohomology. In the case where the basis is no longer supposed to be commutative, de Rham cohomology is replaced by Karoubi-de Rham cohomology, double Poisson algebras were introduced by M. Van den Bergh and their cohomology was defined by Pichereau-Van de Weyer. We will develop a differential calculus over double Lie-Rinehart algebras that will allow us to see in a unified way Karoubi-de Rham cohomology and double Poisson cohomology. □

Francesco D'Andrea, Universitá di Napoli Federico II
CW structures in noncommutative geometry

Abstract: I will illustrate some examples and ideas for a theory of CW complexes in noncommutative geometry. In order to accommodate some important examples, instead of diagrams in the category of quantum spaces (dual to C^*-algebras) one is forced to work with a suitable homotopy category. In this category, K-theory computations are made possible through the use of a Mayer-Vietoris sequence. The K-theory of a quantum space can be promoted from a plain abelian group to an augmented ring (in the sense of Chevalley-Eilenberg), giving a finer topological invariant. In order for this construction to be functorial, one has to equip quantum

spaces with a"kind of topology" (a coverage in the sense of Grothendieck). The talk is based on a work in progress with P.M. Hajac, T. Maszczyk, A. Sheu, B. Zielinski. □

Marvin Dippell, Universität Würzburg
Towards an HKR-Theorem for Coisotropic Reduction

Abstract: Deformation quantization aims to construct a quantum analog of a classical phase space given by a Poisson manifold, by deforming the commutative multiplication on the classical observable algebra into a non-commutative, so called, star product. In classical mechanics symmetry reduction plays an important role, and it can be formalized in geometric terms using coisotropic reduction. In this talk I want to introduce a subcomplex of the classical Hochschild complex, called constraint Hochschild complex, which controls deformations compatible with the reduction data. As a first step towards a HKR-Theorem for this subcomplex we will compute the second constraint Hochschild cohomology class in the local situation. In the end I will give a glimpse into a general framework which allows to treat geometric and algebraic objects with reduction data on equal footing and might provide tools for the proof of a HKR-Theorem for coisotropic reduction. □

Fabio Gavarini, Universitá di Roma, Tor Vergata
Multiparameter quantum groups: a unifying approach

Abstract: The original quantum groups — in particular, quantized universal enveloping algebras, in short QUEA's — have been introduced as depending on just one "continuous" parameter. Later on, multiparameter quantum groups — in particular, multiparameter QUEA's — have been introduced in differente ways, with the new, "discrete" parameters either affecting the coalgebra structure or the algebra structure (while leaving the dual structure unchanged). Both cases can be realized as special type deformations — namely, either by twist, or by 2-cocycle deformation — of Drinfeld's celebrated QUEA. In this talk I will introduce a new, far-reaching family of multiparameter QUEA's that encompasses and generalizes the previous ones, while also being stable with respect to both deformations by twists and deformations by cocycles. Taking semiclassical limits, these new multiparameter QUEA's give rise to a new family of multiparameter Lie bialgebras, that in turn is stable under both by twist and deformations by 2-cocycles (in the Lie bialgebraic sense). This is a joint work with Gastón Andrés García — cf. arXiv:2203.11023 (2022). □

Anthony Giaquinto, Loyola University
Schur-Weyl Duality for Braid and Twin Groups

Abstract: The symmetric group admits two natural covering groups: the braid group and the twin group. These are obtained, respectively, by removing the involution and cubic relations in the Coxeter presentation of the symmetric group. There is a natural Bureau representation for each group, which for the braid group is a q-analog of the permutation representation of the symmetric group and for the twin group is a related orthogonal representation generated by complex reflections. New instances of Schur-Weyl duality are found by examining the diagonal action of these groups on tensor powers of the Bureau representation. The centralizer algebra of the action of each group is described diagramatically in terms of partial permutation and partial Brauer algebras. As a result, we obtain many representations of the braid and twin groups that can be combinatorially constructed. This is joint work with Stephen Doty. □

Madeleine Jotz-Lean, Universität Würzburg
A geometrisation of $[n]$-manifolds

Abstract: This talks sketches the equivalence between positively graded manifolds of degree n and symmetric n-fold vector bundles; i.e. n-fold vector bundles equipped with an S_n-symmetry. The degree 2-case is the geometrisation of [2]-manifolds by involutive double vector bundles; or equivalently by metric double vector bundles.

This is joint work with Malte Heuer. □

Niels Kowalzig, Universitá di Napoli, Federico II
Centres, traces, and cyclic cohomology

Abstract: While centres of monoidal categories are the key to Gerstenhaber algebras structures, bimodule category centres allow for cyclic cohomology to be defined and hence are the key to BV algebra structures. In this talk, we will discuss the biclosedness of the monoidal categories of modules over a (left) Hopf algebroid, along with a categorical equivalence between bimodule category centres and anti Yetter-Drinfel'd contramodules. This is directly connected to the existence of a trace functor on the monoidal category in question, which in turn allows to recover (or define) cyclic operators enabling cyclic cohomology. □

Camille Laurent-Gengoux, Université de Lorraine
Dg-manifolds and singular foliations

Abstract: We will present some elementary questions about various geometrical problems, most of them having singular foliations involved, that dg-manifolds help solving. Joint works with Lavau, Louis, Ryvkin and Strobl. □

Leonid Ryvkin
L_∞ **Extensions for the Poisson algebra of a symplectic manifold**

Abstract: Let (M, ω) be a non-compact symplectic manifold. The universal central extension of the Lie algebra $C^\infty(M)$ can be realized as the quotient $\Omega^1(M)/\delta\Omega^2(M)$, where δ is the differential of the canonical homology complex. We will show how the Lie algebra $\Omega^1(M)/\delta\Omega^2(M)$ extends to an L_∞-algebra structure on the canonical homology complex. The extension procedure is inspired by a similar approach for general multisyplectic manifolds developed by Christopher Rogers, but is combinatorially more intricate and relies on results from symplectic Hodge theory. (j.w. Bas Janssens and Cornelia Vizman) □

Stefano Ronchi, Universität Göttingen
Searching for duals of higher VB-groupoids

Abstract: A VB-n-groupoid is an n-groupoid (as a simplicial object satisfying Kan conditions) in the category of vector bundles. It is known that VB-1-groupoids are in correspondence, up to non-canonical isomorphism, with 2-term representations up to homotopy of Lie groupoids. On one hand, this reduces to the classical Dold-Kan correspondence when restricting to representations of the point groupoid, and on the other, it can be generalized to a statement about higher VB-groupoids and representations up to homotopy of (higher) Lie groupoids. In this talk I will show how this approach can provide insight for the problem of defining dual objects for VB-n-groupoids, focusing on the case of n=2, and the main difficulties that appear when passing from the 1-case to the 2-case. I will then present our current progress and possible applications of this result. This is a summary of several projects joint with Miquel Cueca, Madeleine Jotz Lean and Chenchang Zhu. □

Andrea Santi, Universitá di Roma, Tor Vergata
$G(3)$ **supergeometry and a supersymmetric extension of the Hilbert–Cartan equation**

Abstract: "I will report on the realization of the simple Lie superalgebra G(3) as symmetry superalgebra of various geometric structures — most importantly super-versions of the Hilbert–Cartan equation and Cartan‚Äôs involutive system that exhibit G(2) symmetry — and compute, via Spencer cohomology groups, the Tanaka–Weisfeiler prolongation of the negatively graded Lie superalgebras associated with two particular choices of parabolics. I will then discuss non-holonomic superdistributions with growth vector $(2|4, 1|2, 2|0)$ obtained as super-deformations of rank 2 distributions in a 5-dimensional space, and show that the second Spencer cohomology group gives a binary quadric, thereby providing a "square-root" of Cartan's classical binary quartic invariant for (2, 3, 5)-distributions. This is a joint work with B. Kruglikov and D. The. □

Paolo Saracco, Bruxelles ULB

From left ideals two sided coideals to normal hopf ideals in hope algebroids, and groupoids.

Abstract: In a celebrated paper from 1972, Takeuchi exhibited a bijective correspondence between normal Hopf ideals (in a geometric perspective, normal subgroups of affine group schemes) and sub-Hopf algebras of a commutative Hopf algebra over a field k, from which one can deduce a purely algebraic proof of the fact that affine commutative k-group schemes form an abelian category. In this talk, I will report on our progresses in the study of normal Hopf ideals in commutative Hopf algebroids and how they relate with affine groupoid schemes. Starting with the correspondence between comodule subrings and left ideals two-sided coideals in an arbitrary bialgebroid, we will arrive at the one between certain normal Hopf ideals (satisfying a faithful flatness condition) and certain sub-Hopf algebroids of a commutative Hopf algebroid. □

Jonas Schnitzer, Universität Freiburg

The strong Homotopy Structure of Phase Space Reduction in Deformation Quantization

Abstract: A Hamiltonian action on a Poisson manifold induces a Poisson structure on a reduced manifold, given by the Poisson version of the Marsden-Weinstein reduction or equivalently the BRST-method. For the latter there is a version in deformation quantization for equivariant star products, i.e. invariant under the action and admitting a quantum momentum map which produces a star product on the reduced manifold. Fixing a Lie group action on a manifold, one can define a curved Lie algebra whose Maurer-Cartan elements are invariant star products together with quantum momentum maps. Star products on the reduced manifold are Maurer-Cartan elements of the usual DGLA of polydifferential operators. Thus, reduction is just a map between these two sets of Maurer-Cartan elements. In my talk I want to show that one can construct an L_∞-morphism, which on the level of Maurer Cartan elements provides a reduction map. This a joint work with Chiara Esposito and Andreas Kraft (arXiv: 2202.08750) □

Mathieu Stiénon, Penn State University

Formal exponential maps and the Atiyah class of dg manifolds

Abstract: Exponential maps arise naturally in the contexts of Lie theory and smooth manifolds. The infinite jets of these classical exponential maps are related to Poincaré‚ÄìBirkhoff‚ÄìWitt isomorphisms and the complete symbols of differential operators. It turns out that these formal exponential maps can be extended to the context of graded manifolds.

For dg manifolds, the formal exponential maps need not be compatible with the homological vector field and the incompatibility is captured by a cohomology class reminiscent of the Atiyah class of holomorphic vector bundles. Indeed, the space of vector fields on a dg manifold carries a natural L_∞ algebra structure whose binary bracket is a cocycle representative of the Atiyah class of the dg manifold.

In particular, the de Rham complex associated with a foliation carries an L_∞ algebra structure akin to the L_∞ algebra structure on the Dolbeault complex of a Kähler manifold discovered by Kapranov in his work on Rozansky-Witten invariants. □

Alfonso Tortorella, Universidade do Porto

Deformations of Symplectic Foliations via Dirac Geometry and L_∞ Algebras

Abstract: In this talk, based on joint work with Stephane Geudens and Marco Zambon, we develop the deformation theory of symplectic foliations, i.e. regular foliations equipped with a leafwise symplectic form. The main result is that each symplectic foliation is attached with a cubic L_∞ algebra controlling its deformation problem. Indeed, we establish a one-to-one correspondence between the small deformations of a given symplectic foliation and the Maurer–Cartan elements of the associated L_∞ algebra. Further, we prove that, under this one-to-one correspondence, the equivalence by isotopies of symplectic foliations agrees with the gauge equivalence of Maurer–Cartan elements. Finally, we show that the infinitesimal deformations of symplectic foliations can be obstructed. □

Stefan Waldmann, Universität Würzburg

Convergence of Star Products

Abstract: In this talk I will briefly discuss the status quo of the quest for convergence of star products. By now there is an increasing number of (classes of) interesting examples which share some common features. □

Ping Xu, Penn State University

BV_∞ quantization of (-1)-shifted derived Poisson manifolds

Abstract: In this talk, we will give an overview of (-1)-shifted derived Poisson manifolds in the C^∞-context, and discuss the quantization problem. We describe the obstruction theory and prove that the linear (-1)-shifted derived Poisson manifold associated to any L_∞-algebroid admits a canonical BV_∞ quantization. This is joint work with Kai Behrend and Matt Peddie. □