Abstracts

Regular talks

Frédéric Barbaresco
Thales, France

Lie Groups Thermodynamics and Foliations Structures of Metriplectic Flow for Machine Learning on Lie groups and Thermodynamics-Informed Neural Networks

Abstract: In the first part, we will present the theme "Statistical learning on Lie groups" [1,2] which extends statistics and machine learning to Lie groups based on the theory of representations and cohomology of Lie algebra. From Jean-Marie Souriau's works on " Lie Groups Thermodynamics" [4] initiated within the framework of symplectic models of statistical mechanics, new geometric statistical tools have been developed to define probability densities (Gibbs density of Maximum Entropy) on Lie Groups or on homogeneous manifolds for supervised methods, and the extension of the Fisher metric of Koszul Information Geometry for unsupervised methods (in metric spaces).

In a 2nd part, TINNs (Thermodynamics-Informed Neural Networks) [3,5] will be discussed for AI-augmented engineering applications. The geometric structures of TINNs are studied by their metriplectic flow (also called GENERIC flow) modeling non-dissipative dynamics (1st law of thermodynamic: energy conservation) and dissipative dynamics (2nd law of thermodynamic: entropy production). Souriau's Lie Groups Dynamics makes it possible to geometrically characterize the metriplectic flow by a webs structure composed of symplectic foliations and transverse Riemannian foliations. From the symmetries of the problem, the coadjoint orbits of the Lie group generate via the moment map (Noether's theorem geometrization) the symplectic foliation (defined as the entropy level sets, where entropy is an invariant Casimir function on these symplectic leaves). The metric on symplectic leaf is given by the Fisher metric. The dynamics along these symplectic leaf, given by the Poisson bracket, characterizes the non-dissipative dynamics. The dissipative dynamics is then given by the transverse Poisson structure and a metric flow bracket, with an evolution from leaf to leaf constrained by the production of entropy. The transverse foliation is a Riemannian foliation whose metric is given by the dual metric of Fisher (Hessian of Entropy). Genesis of Jean-Marie Souriau ideas and his biography are given in [6].

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Özgür Ceyhan
University of Luxembourg

Tropical Geometry in Backpropagation

Abstract: This work presents tropicalization, a technique that approximates tropical neural networks as limits of deep ReLU networks. Tropicalization converts the initial weights from real numbers to those in the tropical semiring while preserving the network's structure. The study confirms that tropicalization does not compromise the classification capabilities of deep neural networks. Additionally, it proposes a tropical version of backpropagation using tropical linear algebra. In tropical arithmetic, multiplication is replaced by addition, and addition is replaced by the maximum function, potentially reducing algorithmic complexity during training and inference. We illustrate this by simulating the tensor multiplication involved in the feed-forward process of advanced neural network models and comparing the standard and tropical versions. Our benchmarks indicate that tropicalization can speed up inference by 50%. Consequently, tropicalization may significantly reduce the training times of large neural networks.

Benoit Dherin
Google

Why neural networks find (geometrically) simple solutions

Abstract: We will start by defining a notion of geometric complexity for neural networks based on intuitive notions of volume and energy. This will be motivated by the visualization of training sequences in the case of simple 1d neural regressions. Then we will explain why for neural networks the optimization process creates a pressure to keep the network geometric complexity low. Additionally, we will see that many other common heuristics in the training of neural networks (from initialization schemes to explicit regularization strategies) have as a side effect to also keep the geometric complexity of the learned solutions low. We will conclude by explaining how this points toward a preference toward a form of harmonic map built in the commonly used training and tuning heuristics in deep learning.

Remco Duits
Eindhoven University of Technology

Geometric Learning via data-efficient PDE-G-CNNs: Training of Association Fields

Abstract: We consider PDE-based Group Convolutional Neural Networks (PDE-G-CNNs) that generalize Group equivariant Convolutional Neural Networks (G-CNNs). In PDE-G-CNNs a network layer is a set of PDE-solvers. The underlying (non)linear PDEs are defined on the homogeneous space M(d) of positions and orientations within the roto-translation group SE(d) and provide a geometric design of the roto-translation equivariant neural network. The network consists of morphological convolutions with (approximative) kernels solving nonlinear PDEs (HJB equations for max-pooling over Riemannian balls), and linear convolutions solving linear PDEs (convection, fractional diffusion). Our an-

alytic approximation kernels are accurate in comparison to our recent exact PDE-kernels. Common mystifying (ReLU) nonlinearities are now obsolete and excluded. We achieve high data-efficiency of our networks: better classification results in image processing with both less training data and less network complexity. Moreover, we have network interpretability as we train sparse association fields (modeling contour perception in our own visual system).

Rita Fioresi
University of Bologna

The Geometry of Deep Learning

Abstract: In this talk we give a brief overview of the challenging geometrical questions coming from machine learning and more specifically from Deep Learning.

Yaël Frégier
Université d'Artois

Transitioning from mathematics to deep learning

Abstract: In this talk I will describe how I have switched from mathematics to deep learning, giving some details on different research projects in health and chemistry, involving techniques such as Wasserstein GANs, self-supervised learning, Equivariant GNNs, diffusion and transformers. I hope that this talk will trigger interest for further discussions.

Xianfeng David Gu
State University of New York at Stony Brook

A Geometric View of Optimal Transport for Generative Models

Abstract: According to the manifold distribution law, a data set can be treated as a distribution on a low dimensional data manifold embedded in the high dimensional ambient space. Therefore, the main tasks for a deep learning system are to learn the manifold structure and the distribution. The later can be achieved using optimal transport maps.

This talk introduces a geometric view of optimal transport: the Brenier theorem for L^2 cost optimal transport map is equivalent to the Alexandrov theorem in differential geometry. This view leads to a geometric variational approach to solve the optimal transport problem, and the regularity property of the Monge-Ampère equation. The results are directly applied for generative models to explain and solve the mode collapsing issue and the physical mistakes made by Sora.

Tao Luo
Shanghai Jiao Tong University

Structure and Gradient Flow Near Global Minima of Neural Networks

Abstract: In this talk, we will investigate the structure of the loss landscape of two-layer neural networks near global minima. Our focus will be on identifying the set of parameters that recover the target function and characterizing the gradient flows around these parameters. By employing novel techniques, our work uncovers some simple aspects of the complex loss landscape, demonstrating how the model, target function, samples, and initialization affect the training dynamics. Our findings conclude that two-layer neural networks can achieve local recovery with overparameterization. If time permits, we will also extend these results to deep neural networks.

Grégoire Sergeant-Perthuis
LCQB, Sorbonne Université

Optimization over functors and application to Bayesian inference and statistical mechanics.

Abstract: A recent trend in deep learning is on how to use deep learning for data with combinatorial structure that encodes some geometry (geometric deep learning [1]: Graph Neural Networks and its generalization Sheaf Neural Networks [2]). The most recent approaches such as Sheaf Neural Networks [3] rely on functors from a cell complex to the category of finite vector spaces (cellular sheaves) [4]; they were firstly proposed as a richer, more expressive data structure in [5]. In [6], we showed that statistical mechanics [12] can be reframed using functors from partially ordered sets (posets) to the category of vector spaces [7]. This opened the way to their use for extending Bayesian inference to the case where signals are heterogeneous, partial, and with possible inconsistencies, extending graphical models, Markov random fields, factor graphs, and more generally, energy-based modeling [8]. Inference on such functors relies on maximizing a weighted sum of entropies that accounts for possible redundancies and which extends the Bethe and Kikuchi free energies [11, 9, 10]. We arrived independently to a similar conclusion that functors serve as a versatile tool that improves the modeling of a large class of signals.

After motivating the use of functors in data science, we will propose a methodology for optimization over those functors. The question we will answer is the following: 'given a collection of objective functions for each signal, where a signal is associated with an element of the poset; how to build a loss function over all the signals that accounts for redundancies?' We will then propose a message passing algorithm to solve such an optimization problem and explain how it extends Bayesian inference and the Belief Propagation algorithm for novel applications in statistical mechanics and energy-based modeling.

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Jan Slovák
Masaryk University

Geometry of Diffusion Tensor Imaging

Abstract: I shall provide a quick survey of geometric methods employed by three of my doctoral students recently. Our aim was to enhance the approchaes to segmentation and fiber tracking in brain imaging. The tools include approximation of the signal attenuation by higher order tensors and Finsler geometry.

Patrick Tardivel
Université de Bourgogne

The solution path of the Sorted L One Penalized Estimation (SLOPE)

Abstract: The SLOPE estimator is a solution to a convex optimization problem that has the particularity of possessing null components (sparsity) and components that are equal in absolute value (clustering). The number of clusters depends on the regularization parameter of the estimator. This parameter can be chosen as a trade-off between interpretability (with a small number of clusters) and accuracy (with a small prediction error). Finding such a compromise requires computing the solution path, which is the function mapping the regularization parameter to the estimator. During this presentation, I will introduce the solution path of SLOPE as well as a numerical scheme, based on a geometrical approach, to solve this path.

Eliot Tron
ENAC Toulouse

Manifold Learning via Foliations and Knowledge Transfer

Abstract: In this talk, I will describe one way to create a natural foliation on the input space of a trained neural network, leveraging on what the network has learned from the dataset. We use the Data Information Matrix, a variation of the Fisher Information Matrix, to provide a natural geometric structure on the space of data. I will show that this matrix is singular, but only on a measure zero set. Moreover, I will present some potential applications of this theory such as knowledge transfer.

Francesco Vaccarino
Politecnico di Torino

Learning and not learning with topology

Abstract: We present two recent results based on the use of algebraic topology and TDA in machine learning: one is the introduction of a novel loss for image processing, that leverages on Morse theory and Persistence; the other is the finding of a topological obstruction to the training of MLP.

Nicky J. van den Berg
Eindhoven University of Technology

Geodesic Tracking via Data-Driven Geometry for Vascular Tree Tracking

Abstract: Retinal images capture the vessels present on the retina which hold a lot of information on a person's overall health. Therefore, it is valuable to develop techniques that automatically identify and track vessels. In this talk, we focus on tracking. We present a new method in which we directly include the geometry present in retinal images.

We first lift the image to the homogeneous space of positions and orientations $\mathbb{M}_2$. This allows the method to disentangle difficult crossing structures. We calculate the geodesics (shortest curves between given seeds and sinks) by solving the Eikonal equation. We rely on a modified metric tensor field when solving this equation. The modification ensures that the Eikonal equation automatically incorporates the geometry of the underlying image.

We also discuss some experimental results. After which we conclude that relying on a data-driven geometry results in improved tracking results.

Xiaoping Wang
The Chinese University of Hong Kong, Shenzhen

Topology optimization using generative models

Abstract: Topology optimization, which aims to find the optimal physical structure that maximizes mechanical performance, is vital in engineering design applications in aerospace, mechanical, and civil engineering. We introduce a deep generative model, based on diffusion models, to address the structure optimization problem. Combine with the threshold dynamics method, we present a successful frame for the topology optimization.

Olga Zaghen
EPFL

Nonlinear Sheaf Diffusion in Graph Neural Networks

Abstract: Sheaf Neural Networks (SNNs) have recently been introduced to enhance Graph Neural Networks (GNNs) in their capability to learn from graphs. Previous studies either focus on linear sheaf Laplacians or hand-crafted nonlinear sheaf Laplacians. The former are not always expressive enough in modeling complex interactions between nodes, such as antagonistic dynamics and bounded confidence dynamics, while the latter use a fixed nonlinear function that is not adapted to the data at hand. To enhance the capability of SNNs to capture complex node-to-node interactions while adapting to different scenarios, we propose a Nonlinear Sheaf Diffusion (NLSD) model, which incorporates nonlinearity into the Laplacian of SNNs through a general function learned from data. Our model is validated on a synthetic community detection dataset, where it outperforms linear SNNs and common GNN baselines in a node classification task, showcasing its ability to leverage complex network dynamics.

Ferdinando Zanchetta
Università di Bologna

Sheaves and deeper geometric structures on graphs

Abstract: The recent successful introduction of Sheaf Neural Networks (SNNs), a particular class of Graph Neural Networks relying on the theory of cellular sheaves, revealed how it is possible to give a notion of sheaf on an undirected graph that can be very useful for applications. After a mathematical introduction to the topic, I will report on some work in progress aimed at providing a more general framework for SNNs that could allow them to handle more structured geometric structures.

Zhen Zhang
Southern University of Science and Technology

An Enhanced Gromov-Wasserstein Barycenter Method for Graph-based clustering

Abstract: Gromov-Wasserstein Learning (GWL) has recently introduced a framework for graph partitioning by minimizing the Gromov-Wasserstein (GW) distance. Various improved versions stemming from this framework have showcased the state-of-the-art (SOTA) performance on clustering tasks. Building upon GW barycenter, we introduce a novel approach that significantly enhances other GW-based models flexibility by relaxing the target distribution (cluster size) in GWL and using a wide class of positive semi-definite matrices. We then develop an efficient algorithm to solve the resulting non-convex problem by utilizing regularization and the successive upper-bound minimization techniques. The proposed method exhibits the capacity to identify improved partitioning results within an enriched searching space, as validated by our developed theoretical framework and numerical experiments. Real data experiments illustrate our method outperforms the SOTA graph partitioning methods on both directed and undirected graphs.

Communications

Iakovos Androulidakis
National and Kapodistrian University of Athens

Hypoellipticity and the Helffer-Nourigat conjecture

Abstract: A differential operator D on a manifold M is called hypoelliptic when for every $f \in C^\infty(M)$, the PDE $Du = f$ admits a smooth solution u. The existing criteria for hypoellipticity usually involve inequalities which might be hard to compute. In 1979, Helffer and Nourrigat conjectured that hypoellipticity is determined by a certain set of representations, associated with the symmetries of D, in the form of nilpotent Lie algebras. These representations are easily computable by a recursive formula. In this talk we will overview the proof of this conjecture, which we gave together with Omar Mohsen and Robert Yuncken.

Paolo Aschieri
Universitá del Piemonte Orientale

Noncommutative differential geometry and Laplacians on graphs

Abstract: We review the noncommutative differential geometry approach to the study of the differential geometry of graphs. This points to a generalization of the notion of graph Laplacian.

Francesco D'Andrea
Universitá di Napoli Federico II

Optimal (Quantum) Transport and Noncommutative Geometry

Abstract: I will give a general overview of the study of metrics on the state space of a C*-algebra ("quantum" states), generalizing the Kantorovich-Rubinstein distance of transport theory. The key notion is that of a generalized Dirac operator. I'll briefly explain how to modify the framework to deal with sub-Riemannian manifolds, which are relevant in optimal control. Here the algebraic data is the so-called Rumin complex, which allows an algebraic reformulation of the Carnot-Carathéodory distance.

Giuseppe Dito
Université de Bourgogne

$\hbar$-expansion of Wightman distributions

Abstract: We will present a computation of the $\hbar$-expansion of the 2-point Wightman distribution for an interacting scalar quantum field theory induced by a twisted star-product. It turns out to be a well-defined formal series in $\hbar$ with coefficients in the space of tempered distributions $\mathcal{S}'$.

Siqi Fu
Rutgers University-Camden

Deformation of Complex and CR structures

Abstract: Deformation of complex structures has been studied extensively since the classical work of Kodaira and Spencer in the 1950s. In this talk, we will discuss deformation of the Complex-Real (CR) structures and its interplay with stability of the spectrum of the Kohn Laplacian. This talk is based in part on the joint work with H. Jacobowitz and W. Zhu.

Niels Kowalzig
Università di Roma Tor Vergata

Higher structures on homology groups

Abstract: We dualise the classical fact that an operad with multiplication, that is, a family of trees with many inputs and one output plus a special element, leads to cohomology groups which form a Gerstenhaber algebra to the context of cooperads, that is, a family of upside-down trees with one input and many outputs, again equipped with a special element: as a result, a cooperad with comultiplication induces a homology theory that is endowed with a graded cocommutative coproduct which is compatible with a coantisymmetric cobracket in a dual Leibniz sense. Possible applications to machine learning will be hinted at.

Emanuele Latini
Università di Bologna

Higher conformal Yang–Mills energy

Abstract: In this talk we set up and formally solve the Yang–Mills boundary problem on conformally compact manifolds. This yields conformally invariant, higher order generalizations of the Yang–Mills equations and their corresponding energy functionals.

Mathieu Stienon
The Pennsylvania State University

Formal geometry of groupoids

Abstract: I will give a brief survey of the formal geometry of groupoids.

Maosong Xiang
Huazhong University of Science and Technology

Cohomology of regular Courant algebroids

Abstract: Differential graded (dg for short) manifolds (a.k.a. Q-manifolds) emerged from a number of areas of mathematics and theoretical physics such as string theory, Hamiltonian mechanics, and derived geometry. Hamiltonian systems in symplectic graded manifolds encodes many interesting geometrical structures. For example, Courant algebroids, introduced by Liu, Weinstein and Xu, can be realized as Hamiltonian systems in symplectic graded manifolds of degree 2. Meanwhile, cohomology of Courant algebroids, defined via dg geometry, plays an important role in the AKSZ's construction of 3D topological Courant sigma models. For each regular Courant algebroid, we construct a minimal model and a Hodge-to-de Rham type spectral sequence to compute its cohomology. This is a joint work with X. Cai and Z. Chen.
