SYMMETRY, DISCRETE GEOMETRY, AND MATHEMATICAL PHYSICS

Dedicated to Adrian Ocneanu

on the occasion of his 70th birthday

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Abstracts

Martin Bojowald
The Pennsylvania State University

Geometry in quantum mechanics

Abstract: Quantum mechanics is amenable to several geometrical formulations. This talk introduces a new version that requires a generalization of Poisson geometry for modern applications of quantum information theory.

Jacob Canel
The Pennsylvania State University

Combinatorics and the Growth of Kac-Moody Algebras

Abstract: Kac-Moody Algebras are much studied, usually infinite dimensional Lie Algebras defined using a generalized Cartan Matrix, extending the definition for semi-simple Lie Algebras. We will demonstrate a visual calculus for constructing their elements and encoding their relations. As an application, we will demonstrate the well known trichotomy theorem about their growth rates.

Robert Coquereaux
Aix-Marseille Université

Intertwiners, honeycombs, and O-blades

Abstract: Within the framework of representation theory of Lie groups of type SU(N) we shall recall the pictograph approach relating Littlewood-Richardson (LR) coefficients to honeycombs and other dual presentations, in particular O-blades, standing for Ocneanu blades. We shall also present a pictographic approach of Kostka coefficients using degenerated O-blades. If times allows, we shall see how the asymptotic behavior of LR coefficients is related to the volume function of the hive polytope, which can be obtained in terms of the Fourier transform of a convolution product of Harish-Chandra's orbital measures, and see how such considerations can be generalized to study intertwiners for other

types of Lie groups.

Yuji Hirota
Azabu University

On Poisson reduction of hamiltonian Lie algebroids

Abstract: Poisson reduction which goes back to the work by J. E. Marsden and T. Ratiu in 1986 is a famous method to obtain new Poisson structures from Poisson manifolds. It contains the Marsden-Weinstein theorem in symplectic geometry, which states that the orbit space of the zero locus of a momentum map by hamiltonian action on a symplectic manifold has a symplectic structure.

Recently, generalization of momentum maps is suggested newly by C. Blohmann and A. Weinstein, which is named *momentum sections*. Momentum sections are defined in terms of Lie algebroids with vector bundle connections satisfying some conditions, and such Lie algebroids is called *Hamiltonian Lie algebroids*. In this talk, after taking a brief look at the study of momentum sections, we shall address reduction problem for Hamiltonian Lie algebroids over Poisson manifolds and show that the zero locus of a momentum section with a certain condition is reducible to a Poisson manifold.

Niels Kowalzig
Università di Roma Tor Vergata

Brackets and products from centres in extension categories

Abstract: Building on Retakh's approach to Ext groups through categories of extensions, Schwede reobtained the well-known Gerstenhaber algebra structure on Ext groups over bimodules of associative algebras both from splicing extensions (leading to the cup product) and from a suitable loop in the categories of extensions (leading to the Lie bracket). We show how Schwede's construction admits a vast generalisation to general monoidal categories with coefficients of the Ext groups taken in (weak) left and right monoidal (or Drinfel'd) centres. In case of the category of left modules over bialgebroids and coefficients given by commuting pairs of braided (co)commutative (co)monoids in these categorical centres, we provide an explicit description of the algebraic structure obtained this way, and a complete proof that this leads to a Gerstenhaber algebra is then obtained from an operadic approach. This, in particular, considerably generalises the classical construction given by Gerstenhaber himself. Conjecturally, the algebraic structure we describe should produce a Gerstenhaber algebra for an arbitrary monoidal category enriched over abelian groups, but even the bilinearity of the cup product and of the Lie-type bracket defined by the abstract construction in terms of extension categories remain elusive in this general setting. Joint work with Domenico Fiorenza.

Xiaobo Liu
Peking University

Universal Equations for Gromov-Witten Invariants

Abstract: Relations among tautological classes on moduli spaces of stable curves have important applications in cohomological field theory. For example, relations among psi-classes and boundary classes give universal equations for generating functions of Gromov-Witten invariants of all compact symplectic manifolds. In this talk, I will talk about such relations and their applications to Gromov-Witten theory and integrable systems.

Roberto Longo
University of Rome Tor Vergata

von Neumann algebras and entropy/energy inequalities in QFT

Abstract: I will present my recent Bekenstein-type, entropy/energy ratio local bound in Quantum Field Theory. This bound is model-independent and rigorous and is proved by operator algebraic methods; it follows solely from first principles in the framework of translation covariant, local Quantum Field Theory on the Minkowski spacetime. I will also

mention its generalization for approximately localized charges and certain relations with the Quantum Null Energy Condition (collaboration with S. Hollands).

Marco Manetti
Sapienza University of Rome

Trace forms on curved Lie algebras and semiregularity maps

Abstract: We propose an extension of a result by Bandiera, Lepri and Manetti (arxiv:2111.12985).

Let (L, d, R) be a curved Lie algebra and (C, δ) a complex of vector spaces, over a field of characteristic 0. A trace form is a linear map $\mathrm{Tr}\colon\ \mathrm{Sym}^* L \to C$ of degree 0 such that $\mathrm{Tr} \circ d = \delta \circ \mathrm{Tr}$ and

$$\sum_{i=1}^{n} (-1)^{|z|\,|x_1\cdots x_{i-1}|}\,\mathrm{Tr}(x_1 \cdots [z, x_i] \cdots x_n) = 0$$

for every $z, x_1, \dots, x_n \in L$.

As in the classical Chern–Weyl theory, to every trace form as above we can associate its Chern character

$$\sum_{k\geq 0} \frac{1}{k!}\,\mathrm{Tr}(R^k) \in \bigoplus_i H^{2i}(C)$$

that is invariant under any twisting $R \mapsto R_x = R + dx + [x, x]/2$ by elements of L^1.

Assume now that L has a filtration $L = F_0 L \supset F_1 L \supset \cdots$ of graded Lie ideals such that $R \in F_1 L$, $d(F_p) \subset F_p$ and $[F_p L, F_q L] \subset F_{p+q} L$ for every p, q. Denote by

$$S_N = \sum_k \sum_{p_1+\cdots+p_k \geq N} F_{p_1} L \cdots F_{p_k} L \subset \mathrm{Sym}^* L$$

the induced filtration of ideals on the symmetric algebra.

Motivated by geometric constructions, e.g. the Buchweitz–Flenner semiregularity map, it makes sense to define, for every $k \geq 0$ the morphism of complexes

$$\sigma_k^1\colon \frac{L}{F_1 L} \to \frac{C}{\mathrm{Tr}(S_{k+1})}[2k], \qquad \sigma_k^1(x) = \frac{1}{k!}\,\mathrm{Tr}(R^k x).$$

Theorem. *The above morphism σ_k^1 is the linear component of an L_∞ morphism, canonically defined up to homotopy equivalence. In particular, the maps $\sigma_k^1\colon H^2(L/F_1 L) \to H^{2+2k}(C/\mathrm{Tr}(S_{k+1}))$ annihilate the obstruction space of the deformation functor associated to the DG-Lie algebra $L/F_1 L$.*

Akifumi Sako
Tokyo University of Science

Quantization of Lie-Poisson algebra and IKKT matrix model

Abstract: A quantization of Lie-Poisson algebras is studied. Classical solutions of the mass-deformed IKKT matrix model can be constructed from semisimple Lie algebras whose dimension matches the number of matrices in the model. We consider the geometry described by the classical solutions of the Lie algebras in the limit where the mass vanishes and the matrix size tends to infinity. Lie-Poisson varieties are regarded as such geometric objects. We provide a quantization called "weak matrix regularization" of Lie-Poisson algebras (linear Poisson algebras) on the algebraic varieties defined by their Casimir polynomials. In order to define the weak matrix regularization of the quotient space by the ideal generated by the Casimir polynomials, we take a fixed reduced Gröbner basis of the ideal. The Gröbner basis determines remainders of polynomials. The operation of replacing this remainders with representation matrices of a Lie algebra roughly corresponds to a weak matrix regularization.

To describe this process more concretely, we can summarize it as follows. We consider a Lie algebra $\mathfrak{g}$ satisfying certain conditions, which includes all semisimple Lie algebras. Among corresponding Lie–Poisson algebras, we denote by $A_{\mathfrak{g}}$ the

one obtained by simply endowing the coordinate polynomial ring of Euclidean space with a Poisson structure. Casimir polynomials correspond with Casimir operators of the Lie algebra by the quantization. Let $I(C)$ denote the ideal generated by a certain Casimir polynomial. Then $A_{\mathfrak{g}}/I(C)$ is also obtained as a Lie–Poisson algebra. Let r_f denote the remainder obtained by dividing a certain $f \in A_{\mathfrak{g}}$ by a Gröbner basis of $I(C)$. Let V^{μ} be the representation space of an irreducible representation of the Lie algebra $\mathfrak{g}$. Then the matrix regularization $A_{\mathfrak{g}}/I(C) \to gl(V^{\mu})$ is constructed by $q_{A/I,\mu} := R_{\mu} \circ \rho_{U/I,\mu} \circ q_{U/I}$.

$$\begin{tikzcd}
A_{\mathfrak{g}}/I(C) \arrow[r, "q_{U/I}\in Q"] \arrow[rr, bend left, "q_{\mu}^{pre}\in Q"] \arrow[rrr, bend left, "q_{A/I_{\mu}}\in Q"] & \mathcal{U}_{\mathfrak{g}}[\hbar]/I(C(X)) \arrow[r, "\rho_{U/I,\mu}"] & gl(V^{\mu}) \arrow[r, "R_{\mu}"] & gl(V^{\mu}) \\
\rotatebox{90}{$\in$} & \rotatebox{90}{$\in$} & \rotatebox{90}{$\in$} & \rotatebox{90}{$\in$}
\end{tikzcd}$$

$$[f(x)] = [r_f + h_f] \mapsto [q_U(r_f)] = [\textstyle\sum_I a_I X_{(i_1,\cdots,i_m)}] \mapsto \textstyle\sum_I a_I e^{(\mu)}_{(i_1,\cdots,i_m)} \mapsto \textstyle\sum_{|I|=m<n_{\mu}} a_I e^{(\mu)}_{(i_1,\cdots,i_m)}$$

Here $q_{U/I} : A_{\mathfrak{g}}/I(C) \to \mathcal{U}_{\mathfrak{g}}[\hbar]/I(C(X))$ is a quantization from $A_{\mathfrak{g}}/I(C)$ to enveloping algebra devided by the ideal made from $I(C)$. $\rho_{U/I,\mu}$ is an expression of $\mathcal{U}_{\mathfrak{g}}[\hbar]/I(C(X))$ to $gl(V^{\mu})$, and R_{μ} is a projection operator that restricts the degree of the polynomial ring.

As concrete examples, we construct weak matrix regularization for $\mathfrak{su}(2)$ and $\mathfrak{su}(3)$. In the case of $\mathfrak{su}(3)$, we not only construct weak matrix regularization for the quadratic Casimir polynomial, but also construct weak matrix regularization for the cubic Casimir polynomial.

This talk is based on the joint work with Jumpei Gohara (TUS): arXiv:2503.24060, arXiv:2205.09019.

Dan-Virgil Voiculescu
UC Berkeley

A hydrodynamic exercise in free probability: free Euler equations

Abstract: The Euler equations for a flow which preserves the Gaussian measure on Euclidean space can be translated in terms of Gaussian random variables , which raises the question about an analogue in free probability. We derive these "free" Euler equations by applying the approach of Arnold for Euler equations to a Lie algebra of infinitesimal automorphisms of the von Neumann algebra of a free group. We then extend the equations to non-commutative vector fields satisfying certain weaker non-commutative smoothness conditions. We also introduce a cyclic vorticity and show that it satisfies appropriate vorticity equations and that it gives rise to a family of conserved quantities.