Application of Nambu bracket to M-theory

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based on
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with P.-M. Ho, Y.Imamura, S.Shiba, C.-S.Chu

- In 2008, D. Berman came to Conference on Noncommutative geometry at Zushi and gave an exciting talk on Bagger-Lambert theory on mutiple M2 branes

- This talk is triggered by his talk and I really appreciate him and organizers to give me such opportunity

Introduction

basics of M-theory

M-theory

- M-theory is yet mysterious theory which unifies 5 superstring theory in 11d
- It consists of two types of branes, M2 and M5 which are dual to each other: by dimensional reduction they reproduce various branes in superstring: F1, NS5, Dp. So M-branes should have some unifying properties of branes of string theory.
- M-theory has no tunable parameters. In this sense, it describes strongly coupled system
- Construction of field theory on multiple M-branes (especially M5) is still the biggest challenge in string theory

D-brane in string theory

- multiple D-brane is described by U(N) SUSY gauge theory which is dimensionally reduced from 10d
- gauge symmetry is described by Lie algebra defined by commutation relation

$$[T^a, T^b] = f^{ab}{}_c T^c$$

- There are $O(N^2)$ dof which represents two ends of open string

Strange statistics of M-brane

= internal degree of freedom

- DOF for N particles or F1 is O(N)
- DOF for N D-brane is O(N^2) -- described by U(N) group
- DOF for N M2-brane is O($N^{3/2}$)
- DOF for N M5-brane is O(N^3)
- What is the symmetry behind this?

AdS/CFT,
anomaly cancellation
scattering of scalar field

Nambu bracket

Generalization of Poisson bracket

$$\{f, g\} = \frac{\partial(f, g)}{\partial(p, q)} \to \{f, g, h\} = \frac{\partial(f, g, h)}{\partial(p, q, r)}$$

Nambu-Poisson bracket

Hamiltonian dynamics

$$\dot{o} = \{H, o\} \to \dot{o} = \{H, K, o\}$$

Time development is described by two Hamiltonians H, K

Description of Symmetry by Nambu bracket

Generalization of Lie algebra: Lie 3-algebra

$$[T^a, T^b] = if^{ab}{}_c T^c \to [T^a, T^b, T^c] = if^{abc}{}_d T^d$$

generalized gauge symmetry

$$\delta\Phi = \lambda_a(x)[T^a, \Phi] \to \delta\Phi = \Lambda_{ab}(x)[T^a, T^b, \Phi]$$

Fundamental identity

Generalization of Jacobi for Lie algebra

$$\{F_1, F_2, \{G_1, G_2, G_3\}\} = \{\{F_1, F_2, G_1\}, G_2, G_3\} + \{G_1, \{F_1, F_2, G_2\}, G_3\} + \{G_1, G_2, \{F_1, F_2, G_3\}\}$$

It is necessary to generate symmetry
but difficult to implement!

Quantization of Nambu bracket

- Matrix realization (Nambu, Curtright-Zachos)

$$[A_1, A_2, A_3] = \sum_{ijk} \epsilon_{ijk} A_i \cdot A_j \cdot A_k$$

- Cubic matrix (Kawamura...)

$$(A, B, C)_{ijk} = \sum_{l} A_{ljk} B_{ilk} C_{ijl}, \quad [A_1, A_2, A_3] = \sum_{ijk} \epsilon_{ijk} (A_i A_j A_k)$$

- String realization (Taktajhan, open membrane)

$$S = \frac{1}{3} \int d^2\sigma \epsilon_{ijk} X^i \frac{\partial X^j}{\partial \tau} \frac{\partial X^k}{\partial \sigma} + \cdots$$

- Zariski quantization (Dito et. al.)

Many trials but difficult to satisfy FI

BLG model

Bagger and Lambert 0711.0955
Gustavson 0709.1260

Field Content on M2

- 8 transverse directions: 8 scalar fields
- 8 fermion
- No gauge field degree of freedom : should be described by TFT: Chern-Simons Field

J. Schwarz

BLG model

$$L = -\frac{1}{2}\langle D^\mu X^I, D_\mu X^I\rangle + \frac{i}{2}\langle\bar{\Psi}, \Gamma_\mu D^\mu \Psi\rangle$$
$$+\frac{i}{4}\langle\bar{\Psi}, \Gamma_{IJ}[X^I, X^J, \Psi]\rangle - V(X) + L_{CS}$$

$$V(X) = \frac{1}{12}\langle [X^I, X^J, X^K], [X^I, X^J, X^K]\rangle$$

$$L_{CS} = \frac{1}{2}\epsilon^{\mu\nu\lambda}\left(f^{abcd}A_{\mu ab}\partial_\nu A_{\lambda cd} + \frac{2}{3}f^{cda}{}_g f^{efgb}A_{\mu ab}A_{\nu cd}A_{\lambda ef}\right)$$

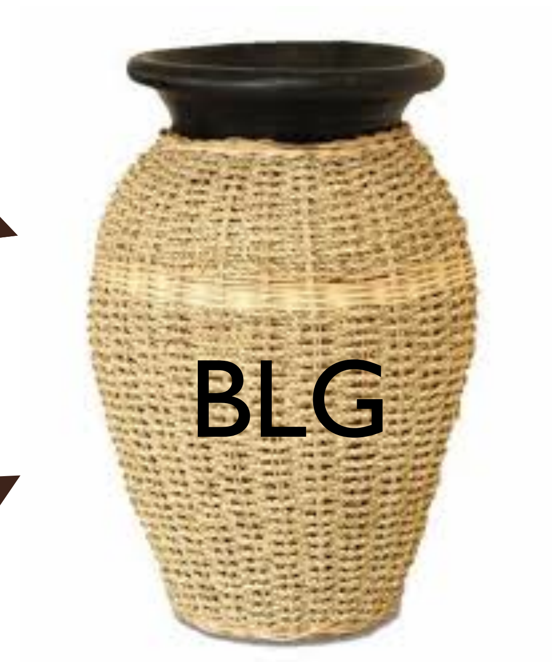
$$X^I = X^I_a T^a \ (I = 1 \sim 8), \quad \Psi = \Psi_a T^a$$
$$(D_\mu X^I)_a = \partial_\mu X^I_a - f^{cdb}{}_a A_{\mu cd} X^I_b$$

N=8 SUSY, No tunable parameters, CFT

Recipe of BLG model

Lie 3 algebra with FI

Positive Invariant metric



N=8 maximal SUSY

3d CFT

$$\langle [T^a, T^b, T^c], T^d \rangle + \langle T^c, [T^a, T^b, T^d] \rangle = 0$$

Generation of huge variety of models in 3D ?

No go theorem

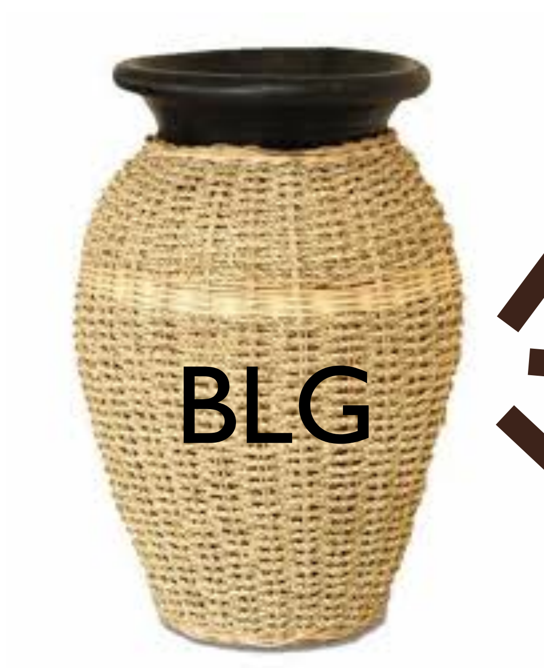
There is only one finite dim.,
3-algebra with positive definite metric

$$A_4 : \quad f^{abc}{}_d = \epsilon_{abcd}\,, \quad h^{ab} = \delta^{ab}$$

HHM(1) Conjecture
Papadopoulos 0804.2662
Gauntlett & Gutowski 0804.3078

Sadly only one model is produced from BLG framework which describes two M2 branes on orbifolds

BLG accepts imperfect 3-algebra after some cook!



ABJM

Giving up anti-symmetry
models with less SUSY
matrix realization

$$[A, B; C] = AB^tC - CB^tA$$

N=6 SUSY

Lorentzian Lie 3

Giving up positivity
Needs Higgs mechanism

Nambu Poisson

Giving up finite dimension
Higher dim branes = M5

Lorentzian metric 3-algebra

Lorentzian metric 3-algebra

T^i : generators of Lie algebra *g*, structure constant $f^{ij}{}_k$

(u, v) : Lorentzian metric generators

$$[v, T^A, T^B] = 0, \quad [u, T^i, T^j] = f^{ij}{}_k T^k, \quad [T^i, T^j, T^k] = -h^{kl} f^{ij}{}_l v,$$

$$\langle u, v \rangle = 1, \quad \langle T^i, T^j \rangle = h^{ij}$$

Essential structure:

v: center of 3-algebra, *u* is not produced by 3-commutator

$g \longrightarrow [g]_3$: a map from arbitrary Lie algebra to 3-algebra

Component expansion

$$X^I = X^I_j T^j + X^I_u u + X^I_v v$$

$$A_\mu = \hat{A}_{\mu i} T^i \wedge u + A'_{\mu jk} T^j \wedge T^k + \cdots$$

X^I_u : "Higgs field" (Mukhi-Papageorgakis)

X^I_v : irrelevant (decoupled from others)

$A'_{\mu ij}$: auxiliary gauge field that is removed after Higgs

$\hat{A}_{\mu i},\ X^I_j$: gauge and scalar field which will survive after Higgs (except for one component in X^I which is absorbed to gauge field)

BLG Lagrangian in terms of components

$$
\begin{aligned}
L &= \Big\langle -\frac{1}{2}(\hat{D}_\mu \hat{X}^I - A'_\mu X^I_i)^2 + \frac{i}{2}\bar{\hat{\Psi}}\Gamma^\mu \hat{D}_\mu \hat{\Psi} + \frac{i}{2}\bar{\Psi}_u \Gamma^\mu A'_\mu \hat{\Psi} \\
&\quad + \frac{i}{2}\bar{\hat{\Psi}}\Gamma_{IJ} X^I_u [\hat{X}^J, \hat{\Psi}] + \frac{1}{4}(X^K_u)^2 [\hat{X}^I, \hat{X}^J]^2 - \frac{1}{2}(X^I_u [\hat{X}^I, \hat{X}^J])^2 \\
&\quad + \frac{1}{2}\epsilon^{\mu\nu\lambda}\hat{F}_{\mu\nu} A'_\lambda \Big\rangle + L_{gh}, \\
L_{gh} &= -\Big\langle \partial_\mu X^I_u A'_\mu \hat{X}^I + (\partial_\mu X^I_u)(\partial_\mu X^I_v) - \frac{i}{2}\bar{\Psi}_v \Gamma^\mu \partial_\mu \Psi_u \Big\rangle,
\end{aligned}
$$

EOM for ghosts are free

$$\partial_\mu^2 X^I_u = 0, \quad \Gamma^\mu \partial_\mu \Psi_u = 0,$$

VEV for ghosts

$$X^I_u = \lambda^I := \lambda \delta^I_{10}, \qquad \Psi^I_u = 0$$

Integrating out A' : One component of X removed (M-direction)
New type of Higgs mechanism (Mukhi-Papageorgakis)

$$L = -\frac{1}{2}(\hat{D}_\mu \hat{X}^I)^2 + \frac{i}{2}\bar{\hat{\Psi}}\Gamma^\mu \hat{D}_\mu \hat{\Psi} + \frac{\lambda^2}{4}[\hat{X}^I, \hat{X}^J]^2 + \frac{i\lambda}{2}\bar{\hat{\Psi}}\Gamma_I [X^I, \hat{\Psi}] - \frac{1}{4\lambda^2}\hat{F}^2_{\mu\nu},$$

Relation with M-theory compactification on S^1

$$\lambda = 2\pi R\,.$$

Higgs VEV gives compactification radius

Decoupling of ghosts can be made exact by *gauged* version:
(Bandres et. al., Gomis et. al.)

Summary:

BLG with $[g]_3$ $\Longleftrightarrow$ Super Yang-Mills with g

Inclusion of Lorenzian generator triggers compactification

Introducing of more (*u*,*v*) pairs

Positive norm generators

$$e^i \quad (i = 1, \cdots, N)$$

Lorentzian norm generators

$$u_a, v_a \quad (a, b = 1, \cdots, M)$$

$$\langle e^i, e^j \rangle = \delta^{ij}\,, \qquad \langle u_a, v_b \rangle = \delta_{ab}\,.$$

$$f^{v_a BCD} = 0$$

v: center, u is not produced by 3-commutators

Non-vanishing structure constant :

Important!

$$\boxed{f^{ijkl} = F^{ijkl},\ f^{aijk} = f_a^{ijk},\ f^{abij} = J_{ab}^{ij},}$$

$$\boxed{f^{abci} = K_{abc}^{i},\ f^{abcd} = L_{abcd}}$$

Here *a*,*b* means u_a, u_b

Useless

Fundamental Identity becomes

$$F^{ijkn}F^{nlmp} + F^{ijln}F^{knmp} + F^{ijmn}F^{klnp} - F^{klmn}F^{ijnp} = 0,$$
$$F^{ijkn}f_a^{nlm} + F^{ijln}f_a^{knm} + F^{ijmn}f_a^{kln} - F^{klmn}f_a^{ijn} = 0,$$
$$f_a^{ijn}F^{nklm} + f_a^{ikn}F^{jnlm} + f_a^{iln}F^{jknm} - f_a^{inm}F^{jkln} = 0,$$
$$(f_a^{ijn}f_b^{nkl} + f_a^{ikn}f_b^{jnl} + f_a^{iln}f_b^{jkn}) + F^{jkln}J_{ab}^{in} = 0,$$
$$J_{ab}^{im}F^{mjkl} + J_{ab}^{jm}F^{imkl} + J_{ab}^{km}F^{ijml} + J_{ab}^{lm}F^{ijkm} = 0,$$
$$(J_{ab}^{im}f_c^{mjk} + J_{ab}^{jm}f_c^{imk} + J_{ab}^{km}f_c^{ijm}) - F^{ijkm}K_{abc}^{m} = 0,$$

$$F^{ijkn}J_{ab}^{nl} - F^{ijln}J_{ab}^{nk} - f_a^{ijn}f_b^{nkl} + f_b^{ijn}f_a^{nkl} = 0,$$
$$(J_{ab}^{im}f_c^{mjk} - J_{ac}^{im}f_b^{mjk}) + (f_a^{ijm}J_{bc}^{mk} - f_a^{ikm}J_{bc}^{mj}) = 0,$$
$$-K_{abc}^{l}f_d^{lij} + K_{abd}^{l}f_c^{lij} + J_{ab}^{il}J_{cd}^{lj} - J_{cd}^{il}J_{ab}^{lj} = 0,$$
$$(f_a^{ikm}J_{bc}^{mi} + f_b^{jkm}J_{ca}^{mi} + f_c^{jkm}J_{ab}^{mi}) + K_{abc}^{m}F^{jkim} = 0,$$
$$(J_{ab}^{jl}J_{cd}^{li} + J_{ad}^{jl}J_{bc}^{li} - J_{ac}^{jl}J_{bd}^{li}) - f_c^{jil}K_{abd}^{l} = 0,$$
$$-J_{ab}^{ki}K_{cde}^{k} - J_{be}^{ki}K_{acd}^{k} + J_{ae}^{ki}K_{bcd}^{k} + J_{cd}^{ki}K_{abe}^{k} = 0,$$
$$f_a^{ijl}K_{bcd}^{l} - f_b^{ijl}K_{acd}^{l} + f_c^{ijl}K_{abd}^{l} - f_d^{ijl}K_{abc}^{l} = 0,$$
$$K_{abc}^{i}K_{def}^{i} - K_{ade}^{i}K_{bcf}^{i} + K_{acf}^{i}K_{bde}^{i} - K_{abf}^{i}K_{cde}^{i} = 0.$$

Solutions of FI

Solution for non-zero F : Nambu Poisson & A_4
For A_4, no nontrivial solution for f, J, K, L
For Nambu Poisson: nontrivial extension
we will see it later

Solution with $F=0$: Classification of Lorenzian Lie algebra
f should be str. const of Lie algebra
J describes (outer) automorphism
nontrivial solutions are given by loop algebra

d-loop algebra: $L^{(d)}g$ *may be possible to add B field*

$$[u_a, u_b] = 0, \quad [u_a, T^i_{\vec{m}}] = m_a T^i_{\vec{m}},$$
$$[T^i_{\vec{m}}, T^j_{\vec{n}}] = m_a v^a \delta_{\vec{m}+\vec{n}} \delta^{ij} + i f^{ij}{}_k T^k_{\vec{m}+\vec{n}} \,.$$

It has d Lorentzian metric pairs: $(u_a, v^b), \quad a = 1, \cdots, d$

Lie 3-algebra version $[L^{(d)}g]_3$ → add (u_0, v^0)

$$[u_0, u_a, u_b] = 0 \,,$$
$$[u_0, u_a, T^i_{\vec{m}}] = m_a T^i_{\vec{m}} \,,$$
$$[u_0, T^i_{\vec{m}}, T^j_{\vec{n}}] = m_a v^a \delta_{\vec{m}+\vec{n}} \delta^{ij} + i f^{ij}{}_k T^k_{\vec{m}+\vec{n}}$$
$$[T^i_{\vec{l}}, T^j_{\vec{m}}, T^k_{\vec{n}}] = -i f^{ijk} \delta_{\vec{l}+\vec{m}+\vec{n}} v^0 \,.$$

More general d-loop 3-algebra

$$
\begin{aligned}
[u_0, u_a, u_b] &= C_{ab} T^0_{\vec{0}} + L_{0abc} v^c, \\
[u_0, u_a, T^i_{\vec{m}}] &= m_a T^i_{\vec{m}} - \delta^i_0 \delta^{\vec{0}}_{\vec{m}} C_{ab} v^b, \\
[u_0, T^i_{\vec{m}}, T^j_{\vec{n}}] &= m_a g^{ij}_{\vec{m}} \delta^{\vec{0}}_{\vec{m}+\vec{n}} v^a + f^{ijk}_{\vec{m}\vec{n}} T^k_{\vec{m}+\vec{n}}, \\
[T^i_{\vec{l}}, T^j_{\vec{m}}, T^k_{\vec{n}}] &= -f^{ijk}_{\vec{l}\vec{m}} \delta^{\vec{0}}_{\vec{l}+\vec{m}+\vec{n}} v^0,
\end{aligned}
$$

C field reflect the effect of noncommutativity
Lie algebra part is given by

$$
\begin{aligned}
[u_a, u_b] &= C_{ab} T^0_{\vec{0}} + L_{0abc} v^c, \\
[u_a, T^i_{\vec{m}}] &= m_a T^i_{\vec{m}} - K^i_{0ab} v^b, \\
[T^i_{\vec{m}}, T^j_{\vec{n}}] &= m_a g^{ij} \delta^{\vec{0}}_{\vec{m}+\vec{n}} v^a + f^{ij}_{\vec{m}\vec{n}}{}^{k} T^k_{\vec{m}+\vec{n}}, \\
[v^a, T^i_{\vec{m}}] &= 0,
\end{aligned}
$$

Simplest example: Kac-Moody algebra

Example of Lorentzian metric Lie algebra : Kac-Moody algebra $L^{(1)}g$

$$
\begin{aligned}
&[u, T^a_m] = mT^a_m, \\
&[T^a_m, T^b_n] = mvg^{ab}\delta_{m+n} + if^{ab}{}_c T^c_{m+n}, \\
&[v, u] = [v, T^a_m] = 0\,,
\end{aligned}
$$

Metric

$$
\langle T^a_m, T^b_n\rangle = g^{ab}\delta_{m+n}, \quad \langle u, v\rangle = 1\,.
$$

Yang-Mills (with world volume *X*)

$$
L \;=\; -\frac{1}{4\lambda^2}\langle F_{\mu\nu}, F^{\mu\nu}\rangle - \frac{1}{2}\langle D_\mu X^I, D^\mu X^I\rangle + \frac{\lambda^2}{4}\langle [X^I, X^J], [X^I, X^J]\rangle
$$

Component expansion

$$
\begin{aligned}
A_\mu &= A_{\mu(a,n)} T^a_n + B_\mu v + C_\mu u, \\
X^I &= X^I_{(a,n)} T^a_n + X^I_u u + X^I_v v, \\
\Psi &= \Psi_{(a,n)} T^a_n + \Psi_u u + \Psi_v v\,.
\end{aligned}
$$

X^I_u : Higgs field

C_μ : flat connection of U(1) bundle

Covariant derivative

$$
\begin{aligned}
(D_\mu X^I)_{(an)} &= \partial_\mu X^I_{an} + f^{bc}{}_a \sum_m A_{\mu(b,m)} X^I_{(c,n-m)} - n C_\mu X^I_{(a,n)} \\
&\quad \underbrace{+ in A_{\mu(a,n)} X^I_u}_{\text{KK mass term}} \Rightarrow F_{\mu y} \quad (y\text{: direction of } X^I) \\
(D_\mu X^I)_u &= \partial_\mu X^I_u;
\end{aligned}
$$

Field strength

$$
\begin{aligned}
(F_{\mu\nu})_{(a,n)} &= \partial_\mu A_{\nu(a,n)} - \partial_\nu A_{\mu(a,n)} + f^{bc}{}_a \sum_m A_{\mu(b,m)} A_{\nu(c,n-m)} \\
(F_{\mu\nu})_u &= \partial_\mu C_\nu - \partial_\nu C_\mu,
\end{aligned}
$$

Role of C_μ

In the covariant derivative, it shows up as,

$$nC_\mu(x)\Phi_{a,n}(x) \Longrightarrow iC_\mu\partial_y\tilde{\Phi}_a(x,y)$$

It describes a U(1) gauge symmetry:

$$\tilde{\Phi}(x,y) \quad \rightarrow \quad e^{i\alpha(x)\partial_y}\tilde{\Phi}(x,y)$$

It implies that the base manifold is not simply X x S^1 but U(1) bundle whose connection is given by C

$$S^1 \rightarrow X$$

EOM

$$\partial^\mu \partial_\mu X_u^I = \Gamma^\mu \partial_\mu \Psi_u = \partial^\mu(\partial_\mu C_\nu - \partial_\nu C_\mu) = 0$$

Gauged version by adding extra gauge fields

$$X_u^I = \text{const.} =: \lambda' \delta^{ID}, \quad \Psi_u = 0, \quad \partial_\mu C_\nu - \partial_\nu C_\mu = 0$$

Dimensional `Oxidation' (or Taylor's T-duality)

$$\tilde{X}_a^I(x,y) = \sum X_{(a,n)}^I(x) e^{-iny/R} \qquad \tilde{A}_{\mu a}(x,y) = \sum_m A_{\mu(a,n)}(x) e^{-iny/R}$$

$$\tilde{X}_a^D(x,y) \to \frac{1}{\lambda} \tilde{A}_{ya}(x,y) .$$

After Higgs, the original action can be described as (d+1)-brane action

$$\frac{1}{2} \int \frac{dy}{2\pi R} \left[\sum_{I=1}^{D-1} (D_\mu \tilde{X}_a^I)^2 + \frac{1}{\lambda^2} \tilde{F}_{\mu y a}^2 \right]$$

Compactification Radius $\quad R = 1/\lambda\lambda'$

Super YM ($L^{(1)}g$, X) ⟺ Super YM(g , $S^1 \to X$)

flat S^1 bundle

Implication to M-theory

Introduction of (d+1)-pairs of Lorentzian generators

M2 → D2 → D(d+2)

Each of Lorentzian pair compactification on S^1

Compactification of M-theory on T^{d+1}

3-algebra automorphism (approximately) represent (part of) U-duality transformation SL(d+1,Z)

$$\vec{\lambda}'^A = \Lambda^A{}_B\, \vec{\lambda}^B \quad \leftrightarrow \quad e'^A = \Lambda^A_B e^B$$

Complete U-duality group

$$SL(d+1;\mathbf{Z}) \bowtie O(d,d;\mathbf{Z}) =: E_{d+1(d+1)}(\mathbf{Z})$$

Nambu-Poisson and M5

Nambu-Poisson as Lie 3

Nambu-Poisson bracket on T^3

$$\{f_1, f_2, f_3\} = \sum_{\dot\mu,\dot\nu,\dot\lambda} \epsilon_{\dot\mu\dot\nu\dot\lambda}\partial_{\dot\mu} f_1 \partial_{\dot\nu} f_2 \partial_{\dot\lambda} f_3 .$$

$$\chi^{\vec{n}}(\vec{y}) = \exp(2\pi i \vec{n}\cdot\vec{y}/R)\,, \quad \chi_{\vec{n}}(\vec{y}) = \exp(-2\pi i \vec{n}\cdot\vec{y}/R)\,.$$

$$h^{\vec{n}_1\vec{n}_2} = \delta(\vec{n}_1 + \vec{n}_2),$$

$$f^{\vec{n}_1\vec{n}_2\vec{n}_3}{}_{\vec{n}_4} = (2\pi i/R)^3 \vec{n}_1\cdot(\vec{n}_2\times\vec{n}_3)\delta(\vec{n}_1+\vec{n}_2+\vec{n}_3-\vec{n}_4)$$

$$f^{\vec{n}_1\vec{n}_2\vec{n}_3,\vec{n}_4} = (2\pi i/R)^3 \vec{n}_1\cdot(\vec{n}_2\times\vec{n}_3)\delta(\vec{n}_1+\vec{n}_2+\vec{n}_3+\vec{n}_4)$$

“Central extension” of NP

Lorentzian generator:

$$u^a = y^a \notin C(T^3)$$

$$\begin{aligned}
f_a^{\vec{n}\vec{m}\vec{l}} &:= \langle \left\{ u^a, \chi^{\vec{n}}, \chi^{\vec{m}} \right\}, \chi^{\vec{l}} \rangle = (2\pi i)^2 \epsilon_{abc} n^b m^c \delta(\vec{n} + \vec{m} + \vec{l}), \\
J_{ab}^{\vec{n}\vec{m}} &:= \langle \left\{ u^a, u^b, \chi^{\vec{n}} \right\}, \chi^{\vec{m}} \rangle = (2\pi i) \epsilon_{abc} n^c \delta(\vec{n} + \vec{m}), \\
K_{abc}^{\vec{n}} &:= \langle \left\{ u^a, u^b, u^c \right\}, \chi^{\vec{n}} \rangle = \epsilon_{abc} \delta(\vec{n}).
\end{aligned}$$

Central extension of Nambu-Poisson by adding v^a *(a=1,2,3)*

$$\begin{aligned}
\left[\chi^{\vec{n}}, \chi^{\vec{m}}, \chi^{\vec{l}} \right] &= F^{\vec{n}\vec{m}\vec{l}}{}_{\vec{p}} \chi^{\vec{p}} - f_a^{\vec{n}\vec{m}\vec{l}} v^a, \\
\left[u^a, \chi^{\vec{n}}, \chi^{\vec{m}} \right] &= f_a^{\vec{n}\vec{m}}{}_{\vec{l}} \chi^{\vec{l}} + J_{ab}^{\vec{n}\vec{m}} v^b, \\
\left[u^a, u^b, \chi^{\vec{n}} \right] &= J_{ab\vec{m}}^{\vec{n}} \chi^{\vec{m}} - K_{abc}^{\vec{n}} v^c, \\
\left[u^a, u^b, u^c \right] &= K_{abc\vec{n}} \chi^{\vec{n}}.
\end{aligned}$$

Field theory on M5

$\mathcal{M}$:original membrane worldvolume x^{μ}

$\mathcal{N}$:3 dim mfd where NP structure is defined $y^{\dot{\mu}}$

M2 (3d) $\Longrightarrow$ M5 (6d)

$\mathcal{M}$ $\quad\quad$ $\mathcal{M} \times \mathcal{N}$

$$X^I_a T^a = X^I_a \chi^a(y) \quad \to \quad X^I(x,y)$$
$$\Psi_a T^a = \Psi_a \chi^a(y) \quad \to \quad \Psi(x,y)$$
$$A_{\mu ab}\chi^a(y)\chi^b(y') \quad \to \quad A_{\mu}(x,y,y')$$

Gauge field bi-local in $y^{\dot{\mu}}$ *?*

Rewriting gauge fields :

$$\begin{aligned} A_{\mu ab} f^{abc}{}_d &= A_{\mu ab} \langle \{\chi^a, \chi^b, \chi^c\}, \chi_d \rangle \\ &= \int_{\mathcal{N}} \epsilon_{\dot{\mu}\dot{\nu}\dot{\lambda}} \frac{\partial}{\partial y^{\dot{\mu}}} \frac{\partial}{\partial y'^{\dot{\nu}}} A_\mu(x, y, y') \Big|_{y'=y} \frac{\partial \chi^c}{\partial y^{\dot{\lambda}}} \chi^d \end{aligned}$$

$$\frac{\partial}{\partial y'^{\dot{\nu}}} A_\mu(x, y, y') \Big|_{y'=y} = \underline{b_{\mu\dot{\nu}}(x, y)}$$

local field in 6d

Division of transverse space

$$I = 1 \sim 8 \quad \begin{cases} \dot{\mu} = \dot{1} \sim \dot{3} & : \quad \text{identify with } \mathcal{N} \\ i = 1 \sim 5 & : \quad \text{transverse direction of M5} \end{cases}$$

We put,

$$X^{\dot{\mu}}(x, y) = y^{\dot{\mu}} + \frac{1}{2}\epsilon^{\dot{\mu}\dot{\nu}\dot{\lambda}} b_{\dot{\nu}\dot{\lambda}}(x, y)$$

X_u is gives radius of T^3..

Fields on M5

$$X^i, \Psi, \underline{b_{\mu\dot{\nu}}, b_{\dot{\mu}\dot{\nu}}}$$

Self-dual two form after field redefinition

it describes field theory on single M5

Truncated NP

Finite dim. version of Nambu Poisson

$\mathcal{H}$: polynomials of x_i $(i=1,2,3)$ with degree $\leq N$

$$\{f,g,h\}_N = \begin{cases} \{f,g,h\} & \text{if deg}\,\{f,g,h\} \leq N \\ 0 & \text{otherwise} \end{cases}$$

It satisfies FI!
Mutually commuting elements: function of two variables
Number of branes: O(N^2)
Number of dof : O(N^3)
Is this an explanation of O($N^{3/2}$) law?
Problem: hard to define the inner product!

Toward multiple M5

cf: Lambert and Papageorgakis

Construction of multiple M5 brane theory

$\longleftrightarrow$ construction of non-abelian self-dual two form

$\longleftrightarrow$ theory of nonabelian gerbe ?

Lambert and Papageorgakis tried to construct such theory by considering SUSY transformation on M5

$$
\begin{aligned}
\delta X^I_A &= i\bar{\epsilon}\Gamma^I\Psi_A \\
\delta\Psi_A &= \Gamma^\mu\Gamma^I D_\mu X^I_A\epsilon + \frac{1}{3!}\frac{1}{2}\Gamma_{\mu\nu\lambda}H^{\mu\nu\lambda}_A\epsilon - \frac{1}{2}\Gamma_\lambda\Gamma^{IJ}C^\lambda_B X^I_C X^J_D f^{CDB}{}_A\epsilon \\
\delta H_{\mu\nu\lambda\, A} &= 3i\bar{\epsilon}\Gamma_{[\mu\nu}D_{\lambda]}\Psi_A + i\bar{\epsilon}\Gamma^I\Gamma_{\mu\nu\lambda\kappa}C^\kappa_B X^I_C\Psi_D f^{CDB}{}_A \\
\delta\tilde{A}_\mu{}^B{}_A &= i\bar{\epsilon}\Gamma_{\mu\lambda}C^\lambda_C\Psi_D f^{CDB}{}_A \\
\delta C^\mu_A &= 0\ ,
\end{aligned}
$$

f should satisfy FI of Lie 3-algebra to make SUSY algebra consistent!

Limitation

eom (= consistency condition of SUSY transformation)

$$
\begin{aligned}
0 &= D^2 X^I_A - \frac{i}{2}\bar{\Psi}_C C^\nu_B \Gamma_\nu \Gamma^I \Psi_D f^{CDB}{}_A - C^\nu_B C_{\nu G} X^J_C X^J_E X^I_F f^{EFG}{}_D f^{CDB}{}_A \\
0 &= D_{[\mu} H_{\nu\lambda\rho]\,A} + \frac{1}{4}\epsilon_{\mu\nu\lambda\rho\sigma\tau} C^\sigma_B X^I_C D^\tau X^I_D f^{CDB}{}_A + \frac{i}{8}\epsilon_{\mu\nu\lambda\rho\sigma\tau} C^\sigma_B \bar{\Psi}_C \Gamma^\tau \Psi_D f^{CDB}{}_A \\
0 &= \Gamma^\mu D_\mu \Psi_A + X^I_C C^\nu_B \Gamma_\nu \Gamma^I \Psi_D f^{CDB}{}_A \\
0 &= \tilde{F}_{\mu\nu}{}^B{}_A - C^\lambda_C H_{\mu\nu\lambda\,D} f^{CDB}{}_A \\
0 &= D_\mu C^\nu_A = C^\mu_C C^\nu_D f^{BCD}{}_A \\
0 &= C^\rho_C D_\rho X^I_D f^{CDB}{}_A = C^\rho_C D_\rho \Psi_D f^{CDB}{}_A = C^\rho_C D_\rho H_{\mu\nu\lambda\,A} f^{CDB}{}_A \ ,
\end{aligned}
$$

implies all fields are constant in the direction specified by C field

$\longleftrightarrow$ could be identified with D4 brane system with non-abelian symmetry

Conclusion

- We review BLG model where Nambu bracket (Lie 3-algebra) plays a fundamental role

- We derived some explicit examples of Lie 3-algebra, particularly focus on central extension in 3-algebra and Nambu-Poisson bracket and its relation with toroidal compactification (with flux)

- BLG model associated with them can be identified with theory with (multiple) D-brane or (single) M5

- U-duality of M-theory can be identified with the automorphism of 3-algebra

- Many issues still remain open
- Is our examples of Lie 3-algebra complete? not likely
- Other branes: NS5, F1 how to derive them?
- BLG (or ABJM) model do not have Lorentz symmetry analog of DBI action?
- theory on multiple M5 is still totally open
- The work by Lambert-Papageorgakis , however, implies that Nambu bracket still plays essentially role