# Algebraic Analysis
## in honor of Masaki Kashiwara's 70th birthday

IHÉS June 6 – 9, 2017

# *Abstracts*

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**Anton Alekseev**
*Université de Genève*

The Kashiwara-Vergne theory and 2-dimensional topology

**Abstract:** The Kashiwara-Vergne problem is a property of the Baker-Campbell-Hausdorff series which was designed to study the Duflo Theorem in Lie theory. Surprisingly, it is related to many other fields of Mathematics including the Drinfled's theory of associators and the theory of multiple zeta values. An interesting new development is a link between the Kashiwara-Vergne theory and 2-dimensional topology encoded in the Goldman bracket and Turaev cobracket on spaces of homotopy classes of loops on surfaces.

The talk is based on joint works with B. Enriquez, N. Kawazumi, Y. Kuno, F. Naef, E. Meinrenken and C. Torossian.

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**Philip Boalch**
*CNRS, Université Paris-Sud*

Nonabelian Hodge spaces and nonlinear representation theory

**Abstract:** The theory of connections on curves and Hitchin systems is something like a "global theory of Lie groups", where one works over a Riemann surface rather than just at a point. We'll describe how one can take this analogy a few steps further by attempting to make precise the class of rich geometric objects that appear in this story (including the non-compact case), and discuss their classification, outlining a theory of "Dynkin diagrams" as a step towards classifying some examples of such objects.

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**Anna Cadoret**
*C.M.L.S., École polytechnique*

Galois representations and invariants in arithmetic geometry

**Abstract:** I will explain how representations of étale fundamental groups can be used to control the degeneration of certain invariants in families of algebraic varieties.

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**Giovanni Felder**
*ETH Zürich*

Elliptic quantum groups and elliptic equivariant cohomology

**Abstract:** I will report on joint work with R. Rimanyi and A. Varchenko. We define an elliptic version of the stable envelope of Maulik and Okounkov for the equivariant elliptic cohomology of cotangent bundles of Grassmannians. It is a version of the construction proposed by Aganagic and Okounkov and is based on weight functions and shuffle products. We obtain an action of the dynamical elliptic quantum groups by difference operators on certain modules over the equivariant elliptic cohomology.

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**Julien Grivaux**
*CNRS, Aix-Marseille Université & IHÉS*

## The Lie algebra attached to a tame closed embedding

**Abstract:** If $X$ is a smooth closed subscheme of an ambient smooth scheme $Y$, Calaque, Căldăraru and Tu have endowed the shifted normal bundle $\mathrm{N}_{X/Y}[-1]$ with a derived Lie algebroid structure. This structure generalizes the Lie algebra structure on the shifted tangent bundle $\mathrm{T}_X[-1]$ on a smooth scheme, due to Kapranov and Markarian. In this talk, we will explain how a geometric condition on the pair $(X, Y)$, originally discovered by Shilin Yu, allows to ensure that $\mathrm{N}_{X/Y}[-1]$ is a true Lie object in the derived category $\mathrm{D}(X)$. Some geometric consequences of this result will be discussed. This is joint work with Damien Calaque.

**Stéphane Guillermou**
*CNRS, Université Grenoble Alpes*

## Microlocal sheaf theory and symplectic geometry

**Abstract:** The microlocal theory of sheaves has been introduced and developed by Kashiwara and Schapira in the 80's, with motivations coming from the theory of $\mathcal{D}$-modules. It has been applied some years ago to the study of symplectic geometry of cotangent bundles in papers of Nadler-Zaslow and Tamarkin. I will explain some results of these papers and subsequent works, in particular how we can associate a sheaf with any Hamiltonian isotopy of a cotangent bundle and how we can use such a sheaf to understand the topology of exact Lagrangian submanifolds.

**Mikhail Kapranov**
*IPMU, The University of Tokyo*

## Perverse schobers on surfaces via Ran categories

**Abstract:** Perverse schobers and conjectural categorical analogs of perverse sheaves, where vector spaces are replaced by triangulated categories. The talk will present a natural definition of such object for the case when the base space is a Riemann surface. The construction is based on a certain analog of the exit path category of the Ran space. Joint work with T. Dyckerhoff, V. Schechtman and Y. Soibelman.

**Masaki Kashiwara**
*RIMS, Kyoto University*

## Riemann-Hilbert correspondence and Laplace transform

**Abstract:** TBA

**Maxim Kontsevich**
*IHÉS, Bures-sur-Yvette*

## Riemann-Hilbert correspondence for $q$-difference modules

**Abstract:** For complex number $q,\ 0 < |q| < 1$ denote by $A_q := \mathbb{C}\langle X^{\pm 1}, Y^{\pm 1}\rangle/(\text{relation } YX = qXY)$ the corresponding quantum torus algebra. By the $q$-version of Riemann-Hilbert correspondence (essentially due to Ramis, Sauloy and Zhang, 2009), the category of holonomic $A_q$-modules is naturally equivalent to the abelian category of coherent sheaves on elliptic curve $E_q := \mathbb{C}^\times/q^{\mathbb{Z}}$ endowed with two anti-Harder-Narasimhan filtrations. I propose a generalization of this correspondence to the higher-dimensional case $A_q^{\otimes n}$, $n > 1$ (joint work in progress with Y. Soibelman). The hypothetical description of the "Betti side" of RH-correspondence combines the categories of constructible sheaves with a given microlocal support, and the bounded derived category of coherent sheaves on the abelian variety $E_q^n$.

**François Loeser**
*Université Pierre et Marie Curie*

## Geometry on arc spaces

**Abstract:** Arc spaces play a central place in recent developments in singularity theory and birational geometry, most notably through motivic integration. The aim of this talk is to present an overview of some recent results about the structure of arc spaces of singular varieties. Taking as a guiding thread a geometric lemma of Denef and myself going back to the early days of motivic integration, I will present some recent work of Reguera, Bouthier - Kazhdan and de Fernex - Docampo.

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**Takuro Mochizuki**
*RIMS, Kyoto University*

## Mixed twistor $D$-modules and some examples

**Abstract:** In the study of mixed twistor $D$-modules, an important issue is to relate mixed twistor $D$-modules with concrete objects in various problems. Although we know that there exist many mixed twistor $D$-modules by an abstract existence theorem and by the functoriality, it is not easy to describe them explicitly, which is one of the blocks to find applications of the theory.

In this talk, after giving a brief survey of the general theory, we shall describe some examples of mixed twistor $D$-modules for which the underlying $\mathcal{R}$-modules can be given explicitly. We shall also mention some examples for which $V$-filtrations can be computed, and we shall explain how they allow us to revisit some known results.

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**Motohico Mulase**
*University of California, Davis, U.S.A.*

## Quantum Curves

**Abstract:** Quantum curves have been conceived in physics as a compact way to encode quantum topological invariants and solutions to certain enumerative geometry problems. In this talk, we start with describing a few concrete mathematical examples. We then present a geometric theory of quantum spectral curves of Higgs bundles. Here, the process of quantization is a passage from moduli of Hitchin spectral curves to families of Rees $\mathcal{D}$-modules on a curve. The talk is based on joint papers with Olivia Dumitrescu.

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**Hiraku Nakajima**
*RIMS, Kyoto University*

## Ring objects in the derived Satake category from Coulomb branches

**Abstract:** In my joint work with Braverman and Finkelberg, we proposed a mathematical definition of Coulomb branches of SUSY gauge theories as Borel-Moore homology of certain varieties which have maps to affine Grassmannians. This construction gives ring objects in derived Satake categories as pushforward of dualizing sheaves in the middle stage. We observe that we could start from ring objects, hence we can define Coulomb branches in more general context.

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**Marco Robalo**
*Université Pierre et Marie Curie*

## Motivic realisations of singularity categories and vanishing cycles

**Abstract:** In this talk I will explain a comparison result establishing an identification of the $\ell$-adic realisation of the $dg$-category of matrix factorisations of a Landau-Ginzburg model over a complete discrete valuation ring with potential induced by a uniformizer, with a 2-periodic version of the inertia-invariant $\ell$-adic vanishing cohomology. This is joint work with A. Blanc, B. Toën, and G. Vezzosi.

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**Raphaël Rouquier**
*UCLA*

## Geometric representation theory as representation-theoretic geometry

**Abstract:** I will explain how varieties occurring in representation theory should be viewed as representations of certain higher categorical structures. This gives a new perspective for the construction and the study of moduli spaces.

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**Takeshi Saito**
*The University of Tokyo*

## Characteristic cycle of an $\ell$-adic sheaf

**Abstract:** The characteristic cycle of an $\ell$-adic sheaf on a smooth variety over a perfect field is a $\mathbf{Z}$-linear combination of irreducible components of the singular support, defined by Beilinson as a closed conical subset of the cotangent bundle. It is an algebraic analogue of that studied by Kashiwara and Schapira in a transcendental setting. We discuss its functorial property with respect to proper direct image.

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**David Treumann**
*Boston College*

## $F$-fields

**Abstract:** An $F$-field on a manifold $M$ is a local system of algebraically closed fields of characteristic $p$. You can study local systems of vector spaces over this local system of fields. On a 3-manifold, they're rigid, and the rank one local systems are counted by the Alexander polynomial. On a surface, they come in positive-dimensional moduli (perfect of characteristic $p$), but they are more stable than ordinary local systems, in the GIT sense. When $M$ is symplectic, maybe an $F$-field should remind you of a $B$-field, it can be used to change the Fukaya category in about the same way. On $S^1 \times \mathbf{R}^3$, this version of the Fukaya category is related to Deligne-Lusztig theory, and I found something like a cluster structure on the Deligne-Lusztig pairing varieties by studying it.

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**Michèle Vergne**
*Université Paris-Diderot*

## The graded equivariant Todd class and the equivariant index of transversally elliptic operators

**Abstract:** Let $G$ be a compact connected Lie group acting on a manifold $M$. Let $\sigma \in K_G(T^*_G M)$ be a transversally elliptic symbol. Thus $Index(\sigma) = \sum_{\lambda \in \hat{G}} m(\sigma, \lambda) V_\lambda$ is a (infinite) sum of irreducible representations $V_\lambda$ of $G$. Let $\mathfrak{t}$ be a Cartan subalgebra for $G$. Considering $\hat{G}$ as a subset of $\mathfrak{t}^*$, we produce a $W$ anti invariant piecewise polynomial function $\xi \mapsto m_{geo}(\sigma, \xi)$ on $\mathfrak{t}^*$, determined by the Chern character of $\sigma$ and the equivariant Todd class, coinciding with $m(\sigma, \lambda)$ on $\hat{G}$. Furthermore, if $M$ is a spin manifold, and $\sigma_k$ is the Dirac operator twisted by a line bundle $L^k$ with proper moment map, we compute the asymptotics when $k \to \infty$ of the distribution $\sum_\lambda m(\sigma_k, \lambda)\delta_{\lambda/k}$ in terms of the formal expansion $\sum_{n=0}^{\infty} Todd_n(M)$ of the equivariant Todd class in the graded equivariant cohomology ring of $M$.

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**Gabriele Vezzosi**
*Università di Firenze*

## Applications of non-commutative algebraic geometry to arithmetic geometry

**Abstract:** We will briefly recall the general philosophy of non-commutative (and derived) algebraic geometry in order to establish a precise link between dg-derived category of singularities of Landau-Ginzburg models and vanishing cohomology, over an arbitrary henselian trait. We will then focus on a trace formula for dg-categories and a recent application to Bloch's conductor conjecture. This second, and main part of the talk refers to work in progress, joint with B. Toën.

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**Tony Yue Yu**
*CNRS, Université Paris-Sud*

## The Frobenius conjecture in dimension two

**Abstract:** We apply the counting of non-archimedean holomorphic curves to the construction of the mirror of log Calabi-Yau surfaces. In particular, we prove the Frobenius structure conjecture of Gross-Hacking-Keel in dimension two. This is joint work with Sean Keel.