International Conference

Relativity and Geometry

in memory of André Lichnerowicz

December 14-16, 2015 Institut Henri Poincaré & Collège de France

Titles and Abstracts

Olivier Biquard
Université Pierre et Marie Curie

On 4-dimensional Einstein manifolds

Abstract: I will describe some recent developments in the theory of 4-dimensional Riemannian Einstein manifolds, mainly about compactness and singularity formation.

Robert Bryant
Duke University

Beyond holonomy: constrained structure equations

Abstract: The problem of determining the possible holonomy groups of G-structures with prescribed conditions has had a long and distinguished history, and the works of A. Lichnerowicz have played a fundamental role in this story.

Many of these questions, at least at the local level, can be understood as problems of determining whether a given set of *structure equations* (in the sense of Élie Cartan) has a solution. More explicitly, given a vector space V of dimension n and a submanifold $A \subset V \otimes \Lambda^2(V^*)$, one wishes to know when there exists an n-manifold M, a V-valued coframing $\omega : TM \to V$, and a mapping $a : M \to A$ satisfying

$$\mathrm{d}\omega = a(\omega \wedge \omega).$$

More generally, one wishes to know how to classify or describe such data (M, ω, a) up to local equivalence. (The case when A is a single point is, of course, resolved by the third fundamental theorem of Lie.) This problem and its natural generalizations (to be discussed in the talk) encompasses a vast array of problems in differential geometry.

I will discuss analysis of this problem via the tools of Cartan-Kähler theory and exterior differential systems and will describe some of its applications to problems in holonomy as well as problems involving geometrically natural PDE that arise in various contexts, including mathematical relativity.

Pierre Cartier
I.H.É.S.

Lichnerowicz and spinors

Abstract: Lichnerowicz seems to be one of the first mathematicians who dealt with spinor fields and the Dirac equation on manifolds. The last sentence of Elie Cartan's book on spinors suggested that the possibility of defining spinor fields on curved Riemannian manifolds was doubtful. Before the great advances of Atiyah and Singer, Lichnerowicz studied the propagators related to the Dirac operator, as well as the square of this operator, and he derived consequences of this study for the topology of Riemannian manifolds. While in the books written by mathematicians, in particular in Chevalley's 1954 book, algebraic methods on Clifford algebras are extensively developed, no mention of the differential geometry aspect of the theory of spinors is to be found. It was Lichnerowicz's work that pioneered the further advances in the geometry of spin manifolds.

Alberto Cattaneo
Universität Zürich

BV-BFV theories on manifolds with boundary and general relativity

Abstract: According to Segal and Atiyah, quantum field theories should be thought of as appropriate functors from a version of the cobordism category. I will describe how this has to be reformulated if one plans to work in perturbation theory, and in particular, how the bulk Batalin-Vilkovisky formalism has to be matched with the boundary Batalin-Fradkin-Vilkovisky formalism. I will then discuss the case of general relativity, both in the Einstein-Hilbert and in the Palatini formalisms.

Thibault Damour
I.H.É.S.

The problem of motion in general relativity

Abstract: The general relativistic problem of motion has acquired a new importance in view of the observations of binary pulsars and of the forthcoming detection of the gravitational wave signals emitted by inspiralling and coalescing compact binary systems. I will review both the history of the general relativistic problem of motion, and the recent analytical and numerical studies of two-body systems. Particular attention will be given to the recently developed "Effective One Body" approach to the motion and radiation of binary systems.

Nathalie Deruelle
A.P.C., Université Paris-Diderot

On the equivalence principle, from Newton to Einstein and beyond

Abstract: I shall ilustrate on a few examples how the "strong" equivalence principle can serve as a guideline to sift gravity theories.

Simon Donaldson
SCGP, SUNY at Stony Brook & Imperial College

Special holonomy and adiabatic limits

Abstract: Riemannian metrics with restricted holonomy provide one of the main sources of examples of compact Einstein manifolds. Many of the holonomy groups that arise can be characterized by the presence of a covariant constant spinor, related to the Lichnerowicz formula for the Dirac operator. The first part of the talk will give a brief general review of this area. In the second part we will discuss "adiabatic limits"—that is, fibred manifolds where the fibre is very small. In particular we will discuss the case of 7-dimensional manifolds with holonomy G_2 and co-asociative fibrations.

Michel Dubois-Violette
Université Paris-Sud

Local Einstein-Cartan actions

Abstract: We describe a large class of local Einstein-Cartan actions for the vacuum Einstein equations. The addition of matter fields actions which involve the connection leads to inequivalent equations which depend on an arbitrary number of parameters.

Charles Frances
Université de Strasbourg

A. Lichnerowicz and the conformal group of pseudo-Riemannian manifolds

Abstract: In the middle of the sixties, A. Lichnerowicz raised the following question: In dimension ≥ 2, is the standard sphere the only compact Riemannian manifold having a noncompact group of conformal transformations? This question was answered positively a few years later by M. Obata and independently J. Ferrand. The aim of the talk is to present those results and their generalizations, as well as the deep impact they had on the study of the automorphism group of geometric structures.

Edward Frenkel
UC Berkeley

Geometry, duality, and quantum physics

Abstract: The geometric Langlands correspondence has been linked to the S-duality of 4D quantum gauge theories in the works of Witten and others. But where does S-duality come from? It turns out that it has a natural geometric explanation from the point of view of a mysterious 6D quantum field theory predicted by string theory and M-theory. Although the existence of this theory and its properties are still conjectural, this information has been used to reveal surprising connections between 4D and 2D quantum field theories, some of which have now been proved. I will review some of these connections and their implications for the geometric Langlands Program, and talk about my recent work with S. Gukov and J. Teschner.

Christian Fronsdal
UCLA

Action principle for thermodynamics?

Abstract: I shall report on efforts to develop an action principle for hydrodynamics and thermodynamics.

1. The original motivation, and other expected benefits of an action principle.
2. A quick review of a special case that is already highly developed, restricted to potential flow, with an application to the problem of the formation of Black Holes.
3. Counting the number of degrees of freedom in hydrodynamics.
4. The most economical way to include vortex motion, with an application to cosmology.
5. Where we stand now and the hope of an advance in electrodynamics in a near future.

Simone Gutt
Université libre de Bruxelles

A symplectic analogue to spaces of constant curvature

Abstract: We describe a class of symmetric symplectic spaces which are natural analogues to Kähler manifolds of constant holomorphic curvature. We describe their totally geodesic submanifolds and show how this yields a framework for Radon-type transforms. This is joint work with Michel Cahen and Thibaut Grouy.

James Isenberg
University of Oregon

Expliciting the space of solutions of the Lichnerowicz equation and the Einstein constraints

Abstract: In 1944, André Lichnerowicz introduced the use of conformal geometry for studying the Einstein constraint equations, which restrict possible choices of initial data for the Einstein gravitational field equations of general relativity. His ideas led to what is now referred to as the "Conformal Method", which to date is the most effective and most widely used technique for finding solutions of the constraint equations, and parametrizing the space of all such solutions. At the heart of the Conformal Method is the "Lichnerowicz equation", which is much like that which he discussed in his 1944 paper. In this talk, after briefly describing Lichnerowicz's early work, we discuss the modern form of the Conformal Method, and explore its success in handling constant mean curvature (CMC) and near-constant mean curvature (near-CMC) initial data sets for the Einstein-vacuum and Einstein-Maxwell theories. We then discuss its challenges and problems in working with initial data sets which are neither CMC nor near-CMC, and in handling the Einstein-scalar constraint equations even in the CMC case.

Sergiu Klainerman
Princeton University

Are black holes real?

Abstract: TBA

Maxim Kontsevich
I.H.É.S.

Noncommutative Kähler geometry

Abstract: By Donaldson-Uhlenbeck-Yau theorem, any stable vector bundle on compact Kähler manifolds admits a unique (up to scalar) Hermitian Yang-Mills connection. I will talk about a potential noncommutative generalization of this result, related to GIT theory, Bridgeland stability and mirror symmetry. This is a work in progress with F. Haiden, L. Katzarkov and P. Pandit.